Math 170, Fall 2008, Section 2
Quiz 4

Name:

1. (10pts) Find the interval of convergence of the series $\sum (-1)^n \frac{(x+2)^n}{n\,2^n}$.

Ratio test: $\left| \frac{(-1)^{n+1}(x+2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n\,2^n}{(-1)^n (x+2)^n} \right| = \frac{n}{n+1} \frac{|x+2|}{2} \xrightarrow[n\to\infty]{} \frac{|x+2|}{2}$

Thus, the series is: convergent for $\frac{|x+2|}{2} < 1$, i.e. $|x+2| < 2$

divergent for $\frac{|x+2|}{2} > 1$, i.e. $|x+2| > 2$.

When $|x+2| = 2$, i.e. when $x = -4$ or $x = 0$, we have:

- $x = -4$: $\sum \frac{(-1)^n (-4+2)^n}{n\,2^n} = \sum \frac{1}{n}$ is divergent

- $x = 0$: $\sum \frac{(-1)^n (0+2)^n}{n\,2^n} = \sum \frac{(-1)^n}{n}$ is convergent

We conclude that the interval of convergence is: $\underline{\underline{(-4, 0]}}$

2. (10pts) Find a power series representation for the function $f(x) = \arctan(x/3)$. Write the answer in the form $\sum_{n=0}^{\infty} c_n x^{2n+1}$. What is the radius of convergence of this power series?

Observe that $f'(x) = \frac{1}{3}\,\frac{1}{1+\left(\frac{x}{3}\right)^2}$. We have the representation:

$$\frac{1}{1+\left(\frac{x}{3}\right)^2} = \frac{1}{1-\left(-\left(\frac{x}{3}\right)^2\right)} = \sum_{n=0}^{\infty}\left(-\left(\frac{x}{3}\right)^2\right)^n = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{3^{2n}},$$ valid for

$\left|-\left(\frac{x}{3}\right)^2\right| < 1$, i.e. $\left|\frac{x}{3}\right| < 1$, or $|x| < 3$.

By the term-by-term integration theorem, the power series representation of f also has a radius of convergence equal to 3, and the representation is:

$$f(x) = \frac{1}{3}\sum_{n=0}^{\infty}\frac{(-1)^n}{3^{2n}}\int x^{2n}\,dx + C = \frac{1}{3}\sum_{n=0}^{\infty}\frac{(-1)^n}{3^{2n}}\,\frac{x^{2n+1}}{2n+1} + C$$

We obtain the value of C by looking at $x=0$: $f(0) = 0 = 0 + C$, i.e. $C = 0$

Finally: $$\boxed{\arctan\left(\frac{x}{3}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\,3^{2n+1}}\,x^{2n+1}}$$

Pledged

Honor code: I have neither given nor received help on this quiz.