

Math 170, Fall 2008, Section 2

Quiz 3

Name:

1. (9pts) Determine whether the series $\sum (3^n + 2^n)/6^n$ is convergent. If it is, find the sum

$$\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}.$$

As the sum of the series

$$\sum \frac{3^m}{6^m} = \sum \left(\frac{1}{2}\right)^m \quad \text{and} \quad \sum \frac{2^m}{6^m} = \sum \left(\frac{1}{3}\right)^m,$$

the series $\sum \frac{3^m + 2^m}{6^m}$ converges.

Moreover, we have:

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n} &= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n \\ &= \frac{1}{2} \times \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n + \frac{1}{3} \times \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n \\ &= \frac{1}{2} \times \frac{1}{1 - \frac{1}{2}} + \frac{1}{3} \times \frac{1}{1 - \frac{1}{3}} = \frac{1}{2-1} + \frac{1}{3-1} \end{aligned}$$

$$\text{Thus: } \boxed{\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n} = \frac{3}{2}}$$

2. (8pts) Find the limit of the sequence $\{\frac{3n}{2n+3}\}$. Does the series $\sum \frac{3n}{2n+3}$ converges?

Observe that $\lim_{n \rightarrow \infty} \frac{3n}{2n+3} = \frac{3}{2}$ $\left(= \lim_{n \rightarrow \infty} \frac{3}{2 + \frac{3}{n}} \right)$

Since $\lim_{n \rightarrow \infty} \frac{3n}{2n+3} \neq 0$,

then necessarily $\sum \frac{3n}{2n+3}$ diverges.

3. (3pts) According to the " ϵ definition" of limits, $\lim_{n \rightarrow \infty} [n^2 - n^3 \sin(1/n)] = 1/6$ means

for every $\epsilon > 0$, there exists an integer N : $n > N \Rightarrow |n^2 - n^3 \sin(1/n) - \frac{1}{6}| < \epsilon$.

Pledged

Honor code: I have neither given nor received help on this quiz.