Math 170, Fall 2008, Section 2
Quiz 2

Name:

1. Find the equation of the tangent line to the curve

$$\begin{cases} x(\theta) &= \tan(\theta), \\ y(\theta) &= \sec(\theta), \end{cases} \qquad -\pi/2 < \theta < \pi/2,$$

at the point corresponding to the value $\theta = \pi/4$.

The derivatives of x and y with respect to θ are:

$$\frac{dx}{d\theta} = \sec^2(\theta) \quad , \quad \frac{dy}{d\theta} = \sec(\theta)\tan(\theta).$$

We obtain: $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sec(\theta)\tan(\theta)}{\sec^2(\theta)} = \frac{\tan(\theta)}{\sec(\theta)} = \sin\theta$$

In particular: $$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \frac{1}{\sqrt{2}}.$$

Besides, we have: $x(\pi/4) = 1$ and $y(\pi/4) = \sqrt{2}$.

The equation of the tangent line is therefore:

$$y - \sqrt{2} = \frac{1}{\sqrt{2}}(x-1) \quad , \text{ or: } y = \frac{1}{\sqrt{2}}(x - 1 + 2)$$

$$\boxed{y = \frac{1}{\sqrt{2}}(x+1)}$$

2. Find the arc length of the curve

$$\begin{cases} x(t) &= e^t + e^{-t}, \\ y(t) &= 1 - 2t, \end{cases} \qquad 0 \leq t \leq 1.$$

You don't need to justify that the curve is traversed only once as t increases from 0 to 1.

The arc length is given by:

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 \sqrt{(e^t - e^{-t})^2 + (-2)^2}\, dt$$

$$= \int_0^1 \sqrt{e^{2t} + e^{-2t} - 2 + 4}\, dt = \int_0^1 \sqrt{e^{2t} + e^{-2t} + 2}\, dt$$

$$= \int_0^1 \sqrt{(e^t + e^{-t})^2}\, dt = \int_0^1 (e^t + e^{-t})\, dt$$

$$= e^t - e^{-t}\Big]_0^1 = (e - e^{-1}) - (1 - 1)$$

$$\underline{L = e - e^{-1}.}$$

Pledged

Honor code: I have neither given nor received help on this quiz.