Math 170, Fall 2008, Section 2
Quiz 1

Name:
Show your work. No work, no credit.

1. **Find the length of the curve**

$$y = \cosh(x), \qquad -1 \leq x \leq 1.$$

Express your answer only in terms of the number e.

The length of the curve is given by:

$$L = \int_{-1}^{1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$= \int_{-1}^{1} \sqrt{1 + \sinh^2(x)}\ dx$$

Observe that $1 + \sinh^2(x) = \cosh^2(x)$ and that $\cosh(x) > 0$ to obtain:

$$L = \int_{-1}^{1} \cosh(x)\, dx = \sinh(x)\Big|_{-1}^{1}$$

$$= \sinh(1) - \sinh(-1) = 2\sinh(1) = 2 \times \frac{e^{1} + e^{-1}}{2}$$

$$\underline{L = e + 1/e}$$

2. Find the arc length function for the curve

$$y = \frac{x^3}{3} + \frac{1}{4x},$$

starting at the point $(1, 7/12)$.

Note that $\frac{dy}{dx} = x^2 - \frac{1}{4x^2}$.

The arc length function is given by:

$$\ell(x) = \int_1^x \sqrt{1 + \left(t^2 - \frac{1}{4t^2}\right)^2}\, dt$$

$$= \int_1^x \sqrt{1 + t^4 - \frac{1}{2} + \frac{1}{16t^4}}\, dt$$

$$= \int_1^x \sqrt{t^4 + \frac{1}{2} + \frac{1}{16t^4}}\, dt$$

$$= \int_1^x \sqrt{\left(t^2 + \frac{1}{4t^2}\right)^2}\, dt = \int_1^x \left(t^2 + \frac{1}{4t^2}\right) dt$$

$$= \frac{t^3}{3} - \frac{1}{4t}\Big|_1^x = \left(\frac{x^3}{3} - \frac{1}{4x}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$\underline{\ell(x) = \frac{x^3}{3} - \frac{1}{4x} - \frac{1}{12}}$$

Pledged

Honor code: I have neither given nor received help on this quiz.