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In Defense of At-Large Representation: A Positive Approach

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I. Introduction

The debate over the relative merits of at-large representation versus single district representation has been a feature of American political life since the founding of the republic. At-large representation has been seen as way to provide a unified set of representatives with a broader political view, but also has the potential for diluting minority interests. Single district representation is perceived as incorporating a greater diversity of views, but risks localism and corrupt “ward” politics. There has, to date, been no attempt to provide a theoretical account that would balance these interests.

At both the federal and state level this has become a moot point. Federal law requires Congressional representation to be chosen in single member districts in all but the most unusual circumstances,¹ and since the 1980’s every state in the union has chosen to apportion its own legislatures in a similar manner.² At the local level, however, it is another story.

While there is a tendency to think of municipalities as having one of two types of governance, either representatives elected at-large or single-district representation, there is a third common method of representation. The mixed system, in which there are both at-large representatives and representatives elected by district, is used in over a fifth of the cities in the United States.³ Among cities with populations between 5,000 and 1,000,000 it is the second most common form of representation (after pure at-large representation) and it accounts for more than half the cities with populations between 250,000 and 500,000.⁴

The percentage of cities using a mixed form of representation peaked in the early 1990’s at close to 30%⁵ as cities moved from systems of pure at-large representation in order to provide more equitable representation to minority voters.⁶ Whether mixed

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¹ *Branch v. Smith* __ US __ (2003).

² At the federal level, one might interpret the representation to the Senate as being “at-large” since both senators are elected statewide. This does raise the question of whether a state should (or even could) apportion itself so as to district its senators. It is an interesting question, but one that I will not address in this paper.

³ Municipal Year Book 1998 40 (Table 5/15).

⁴ *Ibid.*

⁵ *Ibid.* at 38.

⁶ Susan A. MacManus, *Mixed Election Systems: The Newest Reform Structure*, 74 Nat’l. Civic Rev. (1985) reprinted in *Local Government Election Practices, A Handbook for Public Officials and Citizens* 40 (Roger L Kemp, ed. 1999).

systems achieve this end is the subject of some debate.⁷ But what is the justification for having any at-large representation at all? Why not just have single districts? And if we want some at-large representation, how much is desirable? That is, if we take seriously both the merits of at-large representation and the importance of districted representation, how should we balance the two? In particular, can one say anything numerically about the proper ratio of the number of districted representatives to the number of at-large representatives? I argue in this paper in favor of just such a proper ratio.

By employing a new cooperative game analysis I have developed elsewhere,⁸ I will show that, under commonly employed hypotheses, the “right” number of at-large representatives is roughly the square root of the total number of representatives. That is, if the city council has 49 seats, then 7 of them should be for at-large representatives. Much of the rest of this paper will be devoted to explaining and supporting the underlying assumptions of the voting model I use as well as defining what I mean by “right”. The purpose of this analysis, then, is to give a *positive* justification for having at-large representation, independent of the traditional normative concerns.

This paper is organized as follows: In the next section I will discuss some of the normative arguments in favor of at-large representation. While these arguments have been found less credible of late, there has been a long-standing interest in at-large representation for both social and political reasons. Section III will introduce the cooperative game model underlying the analysis. This model is based on Banzhaf’s⁹ which has appeared frequently in legal analysis. In section IV I will define and defend the hypotheses that are inherent in the model and in section V I will discuss what the model maximizes in order to find the “right” answer. Section VI will discuss the positive results of the model.

⁷ The most recent study on the relative merits of at-large, district, and mixed systems is Susan Welch, *The Impact of At-Large Elections on the Representation of Blacks and Hispanics*, 52 J. of Pol. 1050 (1990). This study finds that mixed systems “are not clearly ahead of at-large systems in ensuring equitable black representation. Cities with mixed systems have not improved their black representational equitability to the same degree at-large systems have.” (at 1060) On the other hand, with respect to Hispanic representation Welch finds “Hispanics appear to do somewhat better in cities with both at-large and district elections than in either of the pure types, a finding which confounds expectations,” although the correlations are relatively weak. (at 1065). Earlier studies showed mixed systems to be more effective relative to at-large representation, at least with respect to African-Americans. See MacManus *supra* note ___ at 44. It is worth remembering also that these studies measure descriptive representation (the ability of blacks and Hispanics to elect members of their own race or ethnicity) rather than substantive representation. I can not find any studies that investigate the relationship between substantive representation and type of governmental structure.

⁸ Paul H. Edelman, *Voting Power and At-Large Representation*, ___ *Mathematical Social Sciences* ___.

⁹ John F. Banzhaf III, *Weighted Voting Doesn’t Work: A Mathematical Analysis*, 19 Rutgers L. Rev. 317 (1965).

II. The Value of At-Large Representation¹⁰

Before considering the appropriate balance between at-large and district representation, we must first decide that there is sufficient merit in each that they are in fact worth balancing. As noted earlier, at the federal and state level it has been decided, that whatever benefits at-large representation offers are not sufficient to outweigh the costs. At the local level, though, no such consensus has emerged. So I begin this paper by surveying the putative advantages of at-large representation in order to justify the next step of finding the right balance between at-large and single district representation.

While the current consensus is clearly in favor of electing representatives to a legislative body from single member districts, the use of multi-member districts, at all levels of representation, has a long history in the United States. At-large election of congressional representatives was common until 1842, when federal legislation was enacted to stop the practice.¹¹ Multi-member districts were common in state legislatures through the 1960's¹² and lasted until the 1980's in at least one.¹³ And, as noted above, at-large representation in municipal councils is the norm. Given the persistence of this form of representation, it is worth considering its benefits.

A number of benefits of at-large representation have been suggested. Unfortunately, there is a paucity of empirical work to validate these claims. The empirical work that has been done is old and less than conclusive. Further complicating matters is the question of whether the benefits that do seem to accrue are due to the multi-member nature of the district or the fact that multi-member districts tend to be larger than similarly situated single member districts.¹⁴ I will return at the end of this section to this

¹⁰ In this section I use both “at-large representation” and “multi-member districts” in this section in similar ways, even though they are somewhat different. Typically one refers to an at-large representative if that representative is elected by the voters in the largest relevant geographical area. Thus senators could be considered at-large representatives from their states. Multi-member districts are parts of a larger geographical area that elect more than one representative to a legislative body that governs the whole geographical area. As an example, in the 3rd Congress (1793) the state of Massachusetts had 3 multi-member districts. Two of those districts elected four members each to the House and one of them elected two members.

This distinction between at-large and multi-member depends on the definition of the relevant geographical area. For instance, in the case of the Senate, if the relevant area is the state, then one would say that senators are elected at-large, but if the relevant area is the whole United States, then one would say that each state constitutes a multi-member district electing 2 senators. The distinction is important when some representatives are elected from districts within the geographical area, and others are elected from the whole area.

¹¹ 1842 Reapportionment Act. We'll return to the history of this development later. It is worth noting that at-large representation nevertheless continued in the House through the 91st Congress in 1969. Typically the need for at-large representation occurred when the state was unable to redistrict in time for the next election, although there were instances in which the state just refused to redistrict at all. Even today there are provisions under 2 USC 2a(c) for election of representatives at-large. The question of whether those provisions have been invalidated by 2 USC 2c (mandating that representatives be elected from single member districts) has been raised, but not definitively answered, in *Branch v. Smith* __ US __ (2003).

¹² Ruth C. Silva, *Compared Values of the Single- and the Multi-Member Legislative District*, 17 West. Pol. Q. 504.

¹³ Indiana did not give up its multi-member lower house districting scheme until then.

¹⁴ Silva, supra note __.

lack of empirical work and speculate as to why there has not been more interest in these questions.

The first possible benefit of at-large representation is the strategic advantage in having a unified delegation, which, of course, is only a benefit when multiple entities—such as states—send delegations to a single legislative body, such as Congress. By electing representatives at-large one is more likely to get cohesive delegation that will be more influential in the legislative body to which it belongs. This strategy was deliberately employed by the small states early in the history of the United States. “As Americans soon discovered, the manner of election influenced the partisan composition of a state’s congressional delegation.... Whereas the minority would be overwhelmed by the majority in an at-large election, the minority gained spokesmen through the districting procedure. This difference resulted in more politically unified congressional delegations for states electing by general ticket and more divided delegations for states electing by district.”¹⁵

It was, in fact, the fear of the power of unified delegations that eventually led to their abolition in 1842. In 1840 the state of Alabama switched from districting to at-large voting for their congressional representatives. The small states began to worry that other large states were going to turn to at-large elections, which “would be disastrous for the small states. If all states had at-large elections, the large-state delegations could be as internally unified as the small.... Not only would the small states lose whatever extra influence at-large elections had given them in the lower house, but they would become a completely inconsequential force there.”¹⁶

More recently there have been similar claims about bloc voting by at-large delegations in state legislatures. In *Whitcomb v. Chavis*¹⁷ voters challenged a multi-member district in Indiana’s legislature. One part of the challenge was that the legislators from this multi-member district voted as a bloc, thus giving disproportionate power to the citizens of that district. A table in the opinion¹⁸ gives quite convincing evidence of slate voting by party; the Court acknowledged the bloc voting by the delegation in the legislature, noting “That bloc voting tended to occur is sustained by the record, and defendants’ own witness thought it was advantageous for Marion County’s delegation to stick together.”¹⁹

The unity, and concomitant increase in power of the at-large delegation, is the flip side of the common complaint that at-large elections will have the effect of diluting minority votes. By choosing representatives at-large, the majority will be able to control the election of every representative, thus providing no voice for the minority voter. In the case of the small states in the early days of the union, this was considered a small price to pay in order to increase their influence: better that the majority of the state’s voters could

¹⁵ Rosemarie Zagari, *The Politics of Size* 126 (1987).

¹⁶ *Ibid.* at 130.

¹⁷ 403 US 124 (1971).

¹⁸ *Ibid.* at 134.

¹⁹ *Ibid.* at 147.

wield more power in Congress than that the state's population be fairly represented but weak. For the purposes of this article, in which I will assume that the at-large representatives come from the same large geographic area as the districted representatives, this particular benefit will be irrelevant. Nevertheless, the fact that at-large representatives tend to vote as a bloc will be significant in my model of a municipal election.

There is a long history of debate as to the effect of multi-member districts on the quality of the representation, as well. A number of possible effects of multi-member districts have been suggested, among them a decrease in localism, better qualified candidates, less emphasis on candidate personality and an increased reliance on party affiliation, and greater party discipline.²⁰ In this paper I will focus on two of these issues: the quality of the candidate and the decrease in localism.²¹

The concern that single-member districts will attract less qualified candidates has a long pedigree. "Proponents of the large district frequently allege that the smaller district results in the election of less able legislators. Because the district is smaller, they say, not only are there fewer competent men to choose from but there is also greater opportunity for less able men to be elected."²² In addition, the argument goes, "multiple districts are more likely to recruit men of breadth and eminence – men who may be passed over or repelled by the triviality and localism of single-district politics."²³ In the late 1800's some claimed to have seen the results of this effect:

In New York, politicians of such diverse temperaments as Thurlow Weed and Horace Greeley were agreed that the legislature, especially the senate, had sunk in the quality of its personnel since the adoption of the single-member system. A Pennsylvania governor, pleading with his state's constitutional convention in 1873 not to establish single districts, cited New York as proof that legislatures decline under the one-member system. This convention discussed the matter extensively.²⁴

This is not to say that this side of the argument predominated. The supporters of single-member districts asserted that, because a single member district was likely to be less diverse in its interests, an elected representative would be better able to attend to

²⁰ Silva supra note __ at 506. Howard Hamilton proffered a list of 13 speculations on the effect of multi-member districts to "furnish something to chew on." See Howard Hamilton, *Legislative Constituencies: Single (sic)-Member Districts, Multi-Member Districts, and Floterial Districts*, 20 West. Pol. Q. 321 (1967). Despite the speculation, there seems to be dearth of empirical work.

²¹ I will focus on these two because the others are less germane to the quality and nature of the representation. In particular, the concerns about the effect on political parties, while important, are not centrally related to how the representatives themselves do their job.

²² Silva supra note __ at 508.

²³ Maurice Klain, *A New Look at the Constituencies: The Need for a Recount and a Reappraisal*, 49 Amer. Pol. Sci. R. 1105 (1955).

²⁴ Ibid. at footnote 30.

those interests. In addition, the representative would be better able to develop personal ties with his constituents and thus be less in debt to his party apparatus.²⁵

It is difficult to know who has the better case here. It is hard to see how the size of the district affects the number of competent people available to run. If the density of “able legislators” is constant one shouldn’t expect to see an increase in the quality of the representatives just by moving to multi-member districts. So it would have to be that the appeal of representing a larger constituency might lead to more competent candidates. There is little empirical work on this question, probably because of the difficulty in deciding on an objective definition of “competent.” The most recent study on analyzing the personal differences (as opposed to the policy differences) between representatives at-large and districted representatives shows that at-large representatives tend to have significantly higher educational levels and incomes than district representatives,²⁶ but it is certainly not clear that this would imply a superior ability to represent the interests of the voter.

In local politics the advocacy of at-large representation is seen as part of the Progressive urban reform movement of the turn of the century. Concerned about the prevalence of ward politics and corruption, the reformers sought to “‘rationalize’ and ‘democratize’ city government by substitution of ‘community oriented’ leadership.”²⁷ The principal tools that were used in pursuit of this “rationalization” were instituting commission style government and the introduction of the city manager, the non-partisan election, and at-large election of representatives.²⁸ The reformers argued that election at-large would eliminate corrupt ward politics from the city council and replace it with a council concerned with the broader interests of the city as whole.²⁹

There is a little question that Progressives were not motivated solely by “good government” concerns. Even one of their more sympathetic defenders has noted that “early civic reform was strongly tainted with nativism.”³⁰ Many historians in the 1960’s came to the view that, the rhetoric of the reformers notwithstanding, the principal problem with the ward system was that “[t]he ward system of government especially gave representation in city affairs to lower- and middle-class groups”³¹ rather than to the business leaders of the city. In the wake of vote-dilution cases such as *White v.*

²⁵ These and other arguments in support for single-member districts can be found in Silva, supra note __ at 507, Klain, supra note __ at 1118, Hamilton, supra note __ at 328 and James D. Barnett, *Unitary-Multiple Election Districts*, 39 Amer. Pol. Sci. R. 65 (1945).

²⁶ Susan Welch & Timothy Bledsoe, *Urban Reform and its Consequences* 42 (1988).

²⁷ Robert L. Lineberry & Edmund P. Fowler, *Reformism and Public Policies in American Cities*, 61 Amer. Pol. Sci. Rev. 701-715 (1967).

²⁸ *Ibid.* at 702.

²⁹ Michael D. McDonald & Richard L. Engstrom, *Minority Representation and City Council Electoral Systems: A Black and Hispanic Comparison* in *Ethnic and Racial Minorities in Advanced Industrial Democracies* 127 (Anthony M. Messina, Lusi R. Fraga, Laurie A. Rhodebeck & Frederick D. Wright, eds. 1992).

³⁰ Richard Hofstadter, *The Age of Reform* 177 (1955).

³¹ Samuel P. Hays, *The Politics of Reform in Municipal Government in the Progressive Era* Pac. Northwest Q. 157 (1964) at 160.

*Regester*³² and the passage of the Voting Rights Act, the civic “reforms” of the Progressives are commonly seen as having the primary effect (and perhaps the purpose) of disenfranchising lower-class and minority voters.³³

Motivations aside, the question remains: do officials elected at-large really provide a qualitatively different type of representation than ones elected from districts? Are they more likely to support the “public interest” rather than narrow partisan concerns as the Progressives asserted? Even those primarily concerned with the effect of at-large representation on minority vote dilution observe that this is an open question.³⁴ The empirical literature is thin and inconclusive.³⁵ Welch and Bledsoe show that at-large representatives spend less time on ombudsman activities than district representatives and focus more on citywide and business constituencies.³⁶ On the issue of policy preferences the results are mixed; they note that “no consistent differences were found in policy attitudes by structure type,”³⁷ although they remark earlier that “[t]he only significant difference between those elected by district and at-large was that at-large members favored raising taxes slightly more than those elected by district” which was “counter to our expectations.”³⁸

Another way to try to measure the effect of at-large representation is to see if there is a correlation between “public-minded” spending and at-large representation. This situation is even more complicated than the personal differences since there are inherently competing interests in spending. At-large representation might lead to less public spending because of the decrease in “ward-healing” and log-rolling, or to an increase based on a larger concern for public works projects. Lineberry and Fowler saw

³² 412 US 755 (1973).

³³ As an example of this perspective see Chandler Davidson & George Korbel, *At-Large Elections and Minority Group Representation* in *Minority Vote Dilution* 65 (Chandler Davidson, ed. 1984). (“An examination of the history of reform during the Progressive Era demonstrates that many reformers, belonging to the business classes, introduced at-large elections to wrest control of municipalities from the laboring classes and ethnic minorities. Research on the use of at-large elections since the 1920s indicates that they continued to be introduced or maintained as barriers to minority office holding, especially when disfranchising measures began to come under challenge.” *Ibid.* at 78.) (“Another consequence of the choice of [at-large elections], however, has been documented impressively and forms the basis of the other side of the debate. That is the tendency for racial minorities to be more underrepresented when council elections are conducted at-large rather than with districts, an under-representation which not infrequently takes the form of total exclusion from the council.” McDonald & Engstrom *supra* note __ at 128.) “Thus, the fundamental premise of the reformers was that at-large elections would disadvantage the lower classes: they disadvantage groups that are spatially segregated (such as blacks and lower-class immigrant groups) and they give advantage to well-organized, well-financed groups, such as business and reform groups.” Welch & Bledsoe, *supra* note __ at 8.

³⁴ “Whether decisions made by at-large councils are truly of higher quality than those made by councils elected under a district format is not something readily documented.” McDonald & Engstrom *supra* note __ at 128.

³⁵ There are a couple of early papers that are primarily concerned with the longevity of candidates, and effects on party structure and discipline. See Silva, *supra* note __ and Hamilton, *supra* note __.

³⁶ Welch & Bledsoe, *supra* note __ at 106 (Table 6.1)

³⁷ *Ibid.*

³⁸ *Ibid.* at 88.

the former effect³⁹ but Morgan and Pelissero found little change in taxing and spending levels among cities that instituted municipal reforms.⁴⁰

Because there is little empirical guidance, we are left to make this judgment on other grounds. Since an at-large representative will be elected by the entire municipality, it is logical to assume that she will be responsive to the majority within the city and thus decide issues in agreement with the majority position. Historically this has been the argument put forward for the position that at-large representatives will be sensitive to the wider interests of the polity, an argument not limited to people in the United States:

After Britain's switch to single districts Arthur Balfour complained that "small, persistent, and selfish interests have now too much power." This is "inevitable in single-member constituencies" What is more, critics argue, the representative tends to develop a narrowness of outlook, a preoccupation with local concerns, a passion for the logroll and the porkbarrel. An assembly based on single district, Gambetta was convinced, is a "broken mirror" in which a community cannot see its own image.⁴¹

Even today there seems to be a presumption that at-large representatives have a responsibility to the polity as a whole. As I write this, Nashville, Tennessee is preparing to elect five at-large representatives to the Metro Council. The Nashville Scene, the local alternative newspaper, published endorsements for the at-large seats in an editorial. The editorial characterizes the at-large seats as ones "whose officeholders are charged with taking a broader view of city issues than their district counterparts."⁴²

Taken together, then, the arguments that at-large representation is fundamentally different in kind than districted representation seem persuasive despite the lack of empirical evidence. And, if that is true, then there would seem to be good reason to consider including some at-large representation in a legislative body. Certainly the concerns that having all (or even most) of the representation at-large would tend to disenfranchise minority interests are legitimate⁴³ but they do not imply that there should be no at-large representation at all. The fact that so many municipalities have mixed representation suggests that it provides some benefit. All of which leaves us with the question of how to balance the influence of at-large representation with districted representation. After a brief digression, that will be the focus of the rest of this paper.

³⁹ Lineberry & Fowler, *supra* note __ at 715.

⁴⁰ David R. Morgan & John P. Pelissero, *Urban Policy: Does Political Structure Matter?*, 74 *Amer. Pol. Sci. Rev.* 999 1005 (1980). The reforms in question were at-large representation, city manager, and non-partisan elections.

⁴¹ Klain *supra* note __ at 1118.

⁴² The Nashville Scene, July 31, 2003 at 5.

⁴³ These concerns themselves have a long history. They were raised in the debates in Congress in 1842, and in various state legislators before that. I do not mean to dismiss them, but only to put them in some perspective.

Before beginning my formal analysis I want to examine the question of why there has been so little recent empirical interest in the nature and desirability of at-large representation. I have two hypotheses. The first is that with the increase of power of the federal government starting in the 1960's academic research moved away to focus on it and its relationship with state and local governments. If this is the case, one might expect to see a rekindling of interest in the study of local politics as power devolves back to local constituencies.

Another possibility is that, as mentioned earlier, the vote dilution litigation of the 1970's and 1980's lead to the perception that at-large representation's only purpose was to disenfranchise minority voters. This lead to a large number of papers investigating the effect of at-large representation on minority representation. Having decided that at-large representation is inherently bad there was little reason to study it further. If at-large representation were no longer a feature of democratic life one might have sympathy with this position, but over 80% of all municipalities have some level of at-large representation.⁴⁴ Are they not worth studying at all? Or do people just pretend that they don't exist?

There does seem to be some precedent for pretending that at-large representation is a thing of the past. In 1955 Maurice Klain wrote a paper that debunked claims that multi-member districts in state legislatures were a thing of the past.⁴⁵ He cited a Committee of the American Political Science Association as reporting that “[p]opular election from single-member districts is the prevailing method by which individual legislators are chosen.”⁴⁶ He cited seven then-classic texts as saying that, with few exceptions, “the rule of one member to each district is generally applied,”⁴⁷ and an additional sixteen contemporaneous texts that made similar assertions.

The problem was that these statements were just false. At the time only nine states chose all of their legislators from single-member districts.⁴⁸ Sixteen states elected at least some of their senators from multi-member districts. Thirty six states elected at least some of their house members from multi-member districts. All together more than 45 per cent of the seats in state houses were elected from multi-member districts.⁴⁹ “The multiple district is no departure from the good old days and the tried old ways. It is not, as giants of the type might lead one nowadays to think, a makeshift arrangement inspired by urban agglomerates. Neither the multiple district nor its widespread use is an emergent of recent times. It is the single –member district that stands outside of history, an alien to the past, a new-fangled and upstart thing.”⁵⁰

⁴⁴ Municipal Year Book 1998 at 40 (Table 5/15).

⁴⁵ Klain, supra note __.

⁴⁶ Ibid at 1105.

⁴⁷ This quote is from Charles A. Beard, *American Government and Politics* 577, 1936 but is representative of claims from the other texts cited. See Klain supra note __ at 1105.

⁴⁸ The states were California, Delaware, Kansas, Kentucky, Missouri, Nebraska, New York, Rhode Island and Wisconsin. Klain, supra note __ at 1106.

⁴⁹ Ibid. at 1108.

⁵⁰ Ibid. at 1111.

Klain does not speculate as to why this myth was so well accepted. Evidently the authors were not sufficiently attuned to their political surroundings to realize that what they were asserting was inaccurate. I can only wonder if the same tendency is true today, and that political scientists are so out of touch with local politics that they have mistakenly concluded that at-large representation is no longer an issue at any level of government.

III. Modeling a Municipal Election

Suppose that we have been convinced by the normative arguments in the previous section that having some at-large representation is a good thing but we don't want to elect all of the representatives at-large for fear of diluting some minority interest. How should we decide on the optimum number of at-large representatives? In order to pick a number I will have to do two things: first I will develop a formal model of a municipal election which will be parameterized by the number of at-large representatives. Then I will choose a function to maximize, one that is dependent on the number of at-large representatives. Finding the function's maximum will determine the number of at-large representatives.

The formal model of the election is based on an understanding of how an individual voter can affect the decision of the city council. This is a two-step process: first the voter casts a ballot for her representative, and then the representative gets to cast a ballot within the council. The influence that a voter can wield is achieved by her ability to alter the choice of the elected official who then has the possibility of altering the decision of the council.

Suppose that a voter is able to cast ballots for both a districted representative and the at-large representatives. Because the number of voters in the district will be smaller than in the whole city, the voter will have a greater opportunity to influence the district race than the at-large race. On the other hand, under assumptions that we will consider shortly, the influence of the at-large representatives on council decisions will be larger than that of the district representatives. It is this essential trade-off that is at the core of my analysis: the voter has more influence over the outcome of the district race, but the district representative will not have as large an influence on the outcome of the council vote as the at-large representatives.

What is the connection between this analysis and the number of at-large representatives? The fewer the number of districted representatives, then the larger the population of each district, which means the more difficult it will be for a voter to influence the outcome of the district election. On the other hand, fewer district representatives means a larger at-large contingent, which will have a larger influence over the decisions of the council. If a voter is concerned solely with her own ability to influence the outcome of a council vote, how should she prefer this trade-off be resolved? I will give a very explicit answer to this question in a subsequent section.

But before I can begin the formal model, I will have to make a number of simplifying assumptions about the behavior of the representatives as well as of the voters:

Assumption 1: The at-large representatives vote as a bloc.

This assumption, while clearly not perfect, is consistent with the general view of at-large elections. If the elections are partisan and there is sufficient party discipline, then bloc voting of the delegation is exactly what one would expect to see.⁵¹ The assumption is also consistent with the principal objection to at-large elections: that they result in the dilution of representation of minorities, which is to say that, at least with respect to these issues, the winners of an at-large election will vote as a bloc in opposition to the minority interests. Unfortunately, I know of no empirical studies to support this assumption.

Assumption 2: A voter casts her ballot independently in the at-large election and in the districted election.

Like Assumption 1, I have not seen any empirical justification for this assumption, but it doesn't seem to be unreasonable. Under a Progressive view of the election, the voter will base her vote in the districted election on ward interests narrowly tailored to her location and her at-large vote will be based on global issues affecting the municipality as a whole. Moreover there is evidence that the more local the election the more it is decided by personal qualities of the candidates and less on particular issues, from which evidence we can conclude that voters are acting independently.

Assumption 3: In the district elections there are two candidates and in the at-large elections there are two slates of candidates. In the district election and in the at-large election, voters are as likely to cast their votes for one side as the other.

Given the abstract nature of this election, the assumption that a voter is as likely to vote for one candidate as the other is the only assumption that one could reasonably make. The difference between analyzing a current electoral system *in situ* and designing an abstract electoral system becomes evident here. While there may be evidence of a pattern of voting in elections in a particular municipality (which is what measures of polarization in voting are about) in this more abstract setting of designing an ideal election system there is little reason to suppose a pattern one way or the other.

The assumption that in the district elections there are only two candidates seems plausible as well. Duverger's Law asserts that plurality elections tend to create a two-party system,⁵² and certainly in areas with strong party affiliations this is likely to be the

⁵¹ As an example see *Whitcomb v. Chavis* 403 US 124 (1971) in which a challenge was made to a multi-member district in Indiana. The table at 134 gives quite convincing evidence of slate voting by party and the Court acknowledged the bloc voting by the delegation in the legislature. ("That bloc voting tended to occur is sustained by the record, and defendants' own witness thought it was advantageous for Marion County's delegation to stick together." At 147)

⁵² Maurice Duverger, *Political Parties: Their Organization and Activity in the Modern State* (1954). For more recent commentary on the evidence in support of Duverger's Law see the essays in *Electoral Laws and their Political Consequences* (Bernard Grofman & Aren Lijphart, eds., 1986).

case. It would be nice to have some empirical evidence in support of this assertion, and in particular to know more about how the size of the district affects partisan politics, but there is little evidence extant.

The most problematic of the assertions in Assumption 3 is that in the at-large election there will effectively be two slates of candidates. To the extent that party politics are important this is likely to be the case⁵³ but if the party affiliations are not strong then this may not be very accurate. An additional troubling point is that part of the Progressive reforms were non-partisan elections. Thus, one might expect areas with at-large representation to be less likely to run candidates on party slates.

Although there has been no formal empirical work done on this question, a look at the most recent election for the at-large representatives to the Metro Council of Nashville is illuminating. There were 19 candidates running for 5 at-large positions. The candidates ended up in three groups: 7 of the candidates each received less than 3% of the vote, 7 candidates received between 4 and 7 % of the vote each, and 5 candidates each received between 9 and 11% of the total vote.⁵⁴ If we ignore the 7 candidates who received less than 3% of the vote then the remaining candidates look very much like two distinct slates. I would not want to push this analogy very far, but it gives some modest support to the assumption. Ultimately, we will just have to take this assumption as a given, and recognize it as the cost of working with a formal model.

Based on these assumptions I can now develop a model of the election using techniques from cooperative games. As these methods are not so widely known, I begin with an introduction to weighted voting games and measuring voting power.

IV. Weighted Voting Models

As discussed previously, my analysis will focus on how an individual voter can influence the outcome of a municipal council decision. This influence occurs in a two-step process: the individual voter might determine which candidate is elected and that candidate may be the determining vote in a council deliberation. Thus, there are two things to consider, how a voter can determine the outcome of the election and how a representative can determine the outcome of the deliberation. I will begin with the latter question, modeling the deliberations of the council

The model I will use is based on a standard weighted voting. Weighted voting is a special case of a cooperative game.⁵⁵ In these games, there is a set of *voters*, each of

⁵³ See Whitcomb again.

⁵⁴ Data from the Nashville Election Commission. These outcomes do not decide the final election, however, because of the additional requirement that a winning candidate receive at least 10% of the vote, something achieved only by three candidates. Thus, there will be a run-off election in September, 2003 for the final two at-large seats between the two candidates who received more than 9% (but not the required 10%) and the next two most popular candidates (who each received 6.3% of the vote.)

⁵⁵ Most elementary game theory texts have an introduction to the subject. For example, see G. Owen, *Game Theory*, 3rd ed., 218 (1995), where such games are called weighted majority games.

whom has a vote of a particular *weight* (not necessarily the same for each voter), and a numerical *quota* which is used to tell if a vote has been successful. The collection of voters who vote in favor of some proposition is called a *coalition*. In an election, each voter casts a vote and the weights of the votes cast in favor are added together. If that sum equals or exceeds the quota, then the issue passes; otherwise it fails. Collections of voters whose total weighted vote are sufficient to carry an election are called *winning coalitions* and if a collection does not have enough weighted vote to carry an election it is called a *losing coalition*.

A brief (and classic) example may help to clarify these ideas. In 1958, the Treaty of Rome established the European Economic Community (EEC) and instituted a weighted voting system for the EEC's governance. The members at that time were France, Germany, Italy, Belgium, the Netherlands, and Luxembourg. The three largest countries (France, Germany and Italy) were each given a vote with weight 4, Belgium and the Netherlands had votes of weight 2 and Luxembourg's vote had weight 1. The quota was 12. Thus, if France, Germany and Italy all voted in favor of some proposition, the weighted vote would be at least $4+4+4=12$. The motion would therefore pass, whether or not the other countries voted in favor of it. So the collection of France, Germany, and Italy is a winning coalition. Similarly, a motion would pass if France, Italy, Belgium, the Netherlands and Luxembourg voted in favor ($4+4+2+2+1=13$) and so they too form a winning coalition. On the other hand, the collection of Italy, Belgium and the Netherlands accounts for a total weighted vote of only $4+2+2=8$, which does not meet the quota and hence is a losing coalition.

In my model of a municipal council I will be starting with the weighted voting game that consists of one voter with large weight, corresponding to the bloc of at-large voter-representatives,⁵⁶ and the rest of the voter-representatives will have weight 1. I model the at-large representatives as a single voter-representative with a large weight because I have made the assumption that they will in fact vote as a single bloc (Assumption 1). To be more precise, I will assume that there are a total of n representatives, k of whom will be from districts and $n-k$ will be elected at-large. Thus, in the model I will have one voter-representative with weight $n-k$ and k voter-representatives with weight 1. The goal, ultimately, will be to choose the value of k that will maximize the influence of a voter-citizen on the outcome of a council deliberation.

I want to focus first on the power of individual voter-representatives to affect the outcome of a council vote. Again, using the EEC as an example, we can see that having a vote on an issue is not the same as having influence over its outcome. To see this, consider Luxembourg's situation within the EEC. It has a vote of weight 1. Is there any situation in which Luxembourg's vote could affect the outcome? Suppose Luxembourg voted in favor of some proposition and that proposition passed. In that case, the total

⁵⁶ In weighted voting theory, whoever casts a vote is referred to generically as a voter. In my application, there are two different types of voters; the representatives who vote in the council and the citizens who elect the representatives. I will need to consider both because of the two-step nature of my analysis. In order to keep this as clear as possible when I am talking about the elections in the council I will refer to the voters as voter-representatives and when I discuss the election of the representatives I will refer to the voters as voter-citizens.

weighted vote in favor in such a scenario would have to be at least 13.⁵⁷ So even if Luxembourg were to change its vote, the proposition would still have gotten a sufficient number of votes (12) to pass. In a similar fashion, if Luxembourg voted against a proposition which failed, changing its vote could not cause the proposition to pass. Thus, Luxembourg's vote is irrelevant to the outcome in all cases. Even though it has a vote, Luxembourg will not have any influence over the outcome.⁵⁸

We must next consider how to measure the amount of influence that a particular voter-representative has on the outcome of a council deliberation. In a council election, a particular vote matters if a change in that vote would alter its outcome. To make this idea precise, we will say that a voter-representative *V* is *pivotal* for a particular winning coalition if *V* is in that winning coalition and the same coalition with *V* removed is a losing coalition. That is, a voter-representative *V* is pivotal for a winning coalition if the weighted vote of the coalition (with *V*) meets the quota (and hence is winning) but if the weighted vote of *V* is removed, then it is less than the quota. For example, in a winning coalition consisting of France, Italy, Belgium, the Netherlands and Luxembourg, France is pivotal, since the coalition Italy, Belgium, the Netherlands and Luxembourg is a losing coalition. In that same winning coalition, Luxembourg is not pivotal since France, Italy, Belgium and the Netherlands is still a winning coalition. In fact, our earlier discussion shows that Luxembourg is never pivotal.

In a similar way, we will call *V* pivotal for a losing coalition if *V* is *not* in that losing coalition, but if *V* is added to it, then it is a winning coalition. In our example, the coalition France, Italy, and Belgium is a losing coalition and Germany is pivotal for this losing coalition since the coalition France, Italy, Belgium and Germany is a winning coalition. The Netherlands is pivotal for the same losing coalition. Luxembourg, of course, is not pivotal for any coalition. Generally, *V* is pivotal for a coalition if i) the coalition is winning, *V* is in the coalition, and *V* is pivotal for it, or ii) the coalition is losing, *V* is not in it, and *V* is pivotal for it. In other words, being pivotal means that *V*'s vote matters because if *V* were to change her vote, the outcome would be different.

This leaves us to decide how to aggregate all these instances into some overall measure of the influence of a voter-representative. The most common way to do this is to define a probabilistic model which assigns to each coalition a likelihood of its being formed, and then computes the likelihood of a voter being pivotal. The particular choice of probabilistic model I will use is due to Banzhaf⁵⁹ and leads to what is referred to as the Banzhaf measure. The Banzhaf measure of voting power is based on the assumption that

⁵⁷ This occurs because only Luxembourg has a vote that has odd weight, all the rest being 2 or 4. So, since Luxembourg is assumed to have voted in favor, the total weight must be odd, and since it is larger than 12, it must be at least 13.

⁵⁸ In this example, we are ignoring the influence that Luxembourg might have over the votes of the other countries. The representatives from Luxembourg might well be able to influence the votes of the other members and in that way indirectly affect the outcome even if their vote will have no direct effect.

⁵⁹ John F. Banzhaf, III, *Weighted Voting Does Not Work*. The mathematics of this measure of voting power dates to earlier independent work of Coleman (J. S. Coleman, *Control of Collectivities and the Power of a Collectivity to Act* in Social Choice 269 (B. Lieberman, ed. 1971) but Banzhaf was the first to apply these voting power measures to the law.

each voter will vote independently from the other voters and that each voter is as likely to vote for some proposition as to vote against it (which conveniently accords with Assumption 3). Equivalently, it assumes that every coalition of voters is equally likely. The Banzhaf measure of a voter's power is equal to the probability of that voter being decisive in a vote, assuming that every coalition is equally likely.

This measure is calculated by computing the number of times that voter is pivotal in some winning coalition and dividing by 2^n , where n is the number of voters in the election. This particular power of 2 is chosen since the total number of possible coalitions among n voters is 2^n , and if each coalition is assumed to be equally likely, the probability of any given coalition is $1/2^n$. In our example of the EEC, we can show that France is pivotal in 10 winning coalitions and 10 losing coalitions, and hence the Banzhaf measure of France's voting power is $20/2^6 = 20/64 = .3125$.⁶⁰ Note that the power of a voter depends only on the weight of its vote, so that the power of Italy and Germany will also be $20/64$. The Banzhaf measure for both Belgium and the Netherlands is $12/64 = .1875$, and the Banzhaf measure of Luxembourg is 0.

What is the significance of the Banzhaf measure of a particular voter? It reflects the ability of that voter to affect the outcome of a vote. Technically, it is the probability that if that voter were to change his vote, the outcome of the vote would be different (given the assumption that every coalition is equally likely).

As another example, consider the weighted voting game with one voter of weight 5 and 10 voters of weight 1. This corresponds to the situation in which there is a municipal council with 5 at-large representatives and 10 districted ones. A majority vote in the council corresponds to a quota of 8 in this weighted voting game. A district representative will be pivotal for a winning coalition if that coalition has a total of exactly 8 weighted votes, and she will be pivotal for a losing coalition if that coalition has a total weighted vote of exactly 7. Calculation shows, then, that a districted representative is pivotal in a total of 144 coalitions⁶¹ and hence has Banzhaf measure $144/2048 \approx .07$. The bloc of at-large voters is pivotal for a winning coalition if the total weight of that coalition is between 8 and 12 (since the removal of 5 votes will result in a losing coalition) and it will be pivotal for a losing coalition if the coalition has a total weighted vote between 3 and 7. The number of such coalitions is 1824 ⁶² and hence the Banzhaf measure of the at-large bloc is $1824/2048 \approx .89$. Thus, in this particular council, the at-

⁶⁰ The ten winning coalitions for which France is pivotal are: France, Germany, Italy; France, Germany, Italy, Luxembourg; France, Germany, Italy, Belgium; France, Germany, Italy, Netherlands; France, Germany, Italy, Belgium, Luxembourg; France, Germany, Italy, Netherlands, Luxembourg; France, Italy, Belgium, Netherlands; France, Germany, Belgium, Netherlands; France, Italy, Belgium, Netherlands, Luxembourg; and France, Germany, Belgium, Netherlands, Luxembourg. The ten losing coalition for which France is pivotal are: Germany, Italy; Germany, Italy, Luxembourg; Germany, Italy, Belgium; Germany, Italy, Netherlands; Germany, Italy, Belgium, Luxembourg; Germany, Italy, Netherlands, Luxembourg; Italy, Belgium, Netherlands; Germany, Belgium, Netherlands; Italy, Belgium, Netherlands, Luxembourg; and Germany, Belgium, Netherlands, Luxembourg.

⁶¹ A given district representative is pivotal in 72 coalitions made up solely of other districted representatives, and in 72 coalitions that include the bloc of at-large representatives.

⁶² This computation is somewhat more complicated than the last. For details of the relevant formula see Edelman, *At-Large Lemma* 6.1.

large bloc would have more than ten times the voting power of each district representative.

To summarize, I will be modeling a municipal council vote as a weighted voting game in which there is one voter-representative with a large weight, corresponding to the bloc of at-large representatives, and many voter-representatives of weight 1 corresponding to the districted representatives. The Banzhaf measure of a voter-representative describes the ability of that voter-representative to affect the outcome of a council vote. There are two technical tasks left: I have to model the citizen-voters' ability to affect the outcome of an election for her representative, and then I have to connect that ability with the citizen-voters' ability to affect the outcome of a council deliberation.

Fortunately, modeling the citizen-voter's ability to affect the outcome of the election for her representative is just a special case of a weighted voting model. Suppose there are P citizen-voters, all of whom vote, in a majority election between two candidates. This is a weighted voting game in which there are P voters, each with weighted vote 1 and a quota equal to $(P+1)/2$.⁶³ It can be shown that the Banzhaf measure of a voter in such a game is approximately $\sqrt{2/(\pi P)}$.⁶⁴ That is, the likelihood of a particular voter being pivotal in a majority election involving P voters is inversely proportional to the square root of P . If the population is broken into k equipopulous districts, then in each district there will be P/k voters, and hence the Banzhaf measure of each of the voters in a majority election for a district representative will be approximately

$$\sqrt{2/\left(\pi \frac{P}{k}\right)} = \sqrt{2k/(\pi P)}.$$

Thus the Banzhaf measure of a citizen-voter in an at-large election is inversely proportional to the square root of the population. If that population is broken into k equipopulous districts, then the Banzhaf measure for a citizen-voter in a district election will be larger than the same citizen's at-large Banzhaf measure by a factor of the square root of k . This shows that there is an inverse relationship between a citizen's influence in an at-large elections and her influence in a districted election.

The final task to complete the model is to connect the citizen-voter's ability to affect the choice of her representatives (both districted and at-large) with the representative-voter's ability to affect the outcome of council decision, in order to determine the ability of the citizen-voter to affect the outcome of a council decision. The classical way to handle this is, again, due to Banzhaf.⁶⁵ By Assumptions 2 and 3 we know that an individual citizen-voter effectively casts two ballots, one for her districted representative and one for her at-large slate, and that these votes are assumed to be

⁶³ There is a small technical point to be dealt with here if P is not odd, but it is not really material to the discussion. Interested readers can consult Dan. S. Felsenthal & Moshe Machover, *The Measurement of Voting Power: Theory and Practice, Problems, and Paradoxes* 66 (1998).

⁶⁴ *Ibid.* at 65.

⁶⁵ John F. Banzhaf, *Multi-member Electoral Districts – Do They Violate the “One Man, One Vote” Principle*, 75 Yale L. J. 1309 (1966).

independent. The individual citizen-voter, then, ultimately has two independent ways to affect the outcome of a vote in the municipal council: If that citizen-voter is pivotal in the majority election for her districted representative (which occurs with probability about $\sqrt{2k/(\pi P)}$) and the district voter-representative is pivotal in a council vote, then the individual citizen-voter has power in this “two-stage” voting game and the size of that power is the product of the powers in each stage. The other way a citizen-voter could be pivotal is if she is pivotal in the at-large election (with probability $\sqrt{2/(\pi P)}$) and the at-large voter-representative bloc is pivotal in the council and the measure of power in this two-step process is, again the product of the measures at each stage. The complete measure of the individual citizen-voter’s power is the sum of these two types of power.⁶⁶

Consider how the number of districts figures into the two parts of a citizen-voter’s power. In the part related to the district vote, the likelihood of an individual citizen-voter being pivotal increases with the number of districts. This is because the larger the number of districts the fewer people per district and hence the likelihood of being pivotal increases.

The influence of the number of districts on the power derived from the at-large component is more difficult to describe precisely⁶⁷ but the intuition is easily seen. The fewer the number of districts, the larger the at-large bloc, since I am assuming that the size of the council is held fixed. The larger the bloc, the more likely it is to be pivotal and hence the power measured by the at-large component is inversely related to the number of districts. So as the number of districts increases, the citizen-voter’s influence on the outcome of the districted election increases. On the other hand, an increase in the number of districts means that the influence of the at-large bloc shrinks, which will make that component of the citizen-voter’s influence get smaller. It is this trade-off that I’ll consider when choosing an optimum number of districts.

V. The Optimum

At last I am in a position to define the measure used to compute the optimum number of at-large representatives. As we have seen in the last section, the power of an individual citizen-voter in this voting game is the sum of the power of that voter in her district election with her power in the at-large election. Moreover, the power in the district election is inversely related to the power in the at-large election. My choice for an optimization criterion will be the total power of a voter. I will choose the number of districts that maximizes the voting power of an individual citizen-voter.⁶⁸

⁶⁶ There is more to this story than meets the eye. While Banzhaf made exactly this claim (Ibid.) and it was employed in litigation (see, e.g., *Morris et. al. v. Board of Estimate* 489 US 688 (1989)) the assertion that the power of the citizen-voter is the sum of the two types of power as described does not follow from the mathematical foundations that he set out. The first place this assertion was rigorously shown is in Edelman, supra note ___.

⁶⁷ For such a formula see Ibid., Lemma 6.1 at 11.

⁶⁸ This measure is called the *sensitivity* of the voting game. See Felsenthal & Machover, supra note ___ at 52.

Why choose the power of an individual citizen-voter as the appropriate measure to maximize? From a rational choice perspective, this is a natural thing to do. In most rational choice models, a citizen will only cast a ballot if his expected benefit of doing so exceeds the cost of voting. The expectation of the benefit is dependent on the likelihood that the vote matters, i.e., is pivotal. So, by choosing to maximize the voting power of the individual voter we will be maximizing the turnout in an election, as well.

I would also argue that implicit in the proposed optimization of voting power is the normative trade-off between at-large representation and districted representation. The goal in having at-large representation is to provide for some representatives beholden to the whole jurisdiction, who will not be biased by parochial local issues. On the other hand we may not want so many at-large representatives that local issues can be ignored. By choosing the number of representatives to maximize the power of the individual voters we are allowing the voters to maximize their influence in a way that balances their concerns between the local and the global issues.⁶⁹

The maximization of voting power is not without its downside. As was seen in Florida during the last presidential election, there is a certain inherent error rate in the process of casting a ballot. Choosing to maximize the voting power implies that the effect of erroneous ballots will be amplified as well. Nevertheless, the benefits of providing each voter with her maximum influence over the outcome would seem to outweigh such a draw back.

VI. The Results

Having specified the model and what I am optimizing I can state the principal result:

The power of an individual voter is maximized when the number of at-large representatives is approximately the square root of the total number of representatives.⁷⁰

For example, the Metropolitan Council of Nashville and Davidson County in Tennessee consists of 40 members and this theorem suggests that the optimum number of at-large representatives is 6.⁷¹ In reality, the Council has 5 at-large representatives.

⁶⁹ Another option would be to choose the number of at-large representatives so that the two contributions to power are equalized. That is, we could find the number that balances the power controlled by a voter in the at-large bloc with the power from her districted representative. Empirically this number seems to be close to the optimum value found by maximizing the total power, although these values are not identical and diverge for large sized councils. Unfortunately I have been unable to prove anything conclusive about this value.

⁷⁰ For the technical details of this theorem see Edelman, At-large, Appendix at 11. The actual result only holds when the total number of representatives exceeds 5.

One implication of this result is to give a positive justification of at-large representation. The optimization performed in the model allowed for the possibility of no at-large representation, and the result shows that voting power is maximized when there is some at-large representation. I know of no other positive justification in support of at-large representation. The increase in voting power is not insubstantial on a relative basis. For councils with sizes between 7 and 21 the relative increase of power of the optimal value over a council of only single member districts ranges between 43% and 45%.⁷²

This analysis also allows one to see how the number of at-large representatives should scale if the size of the council changes. That is, if Nashville were to decide to halve the size of the Metro Council, what would be the appropriate number of at-large representatives to include? One thought would be to halve the current number of at-large representatives, but the model would suggest something different. It suggests that the number should drop to about 70% of its previous size.⁷³

The number of at-large representatives prescribed by this theorem is mathematically interesting as well. The square root function arises in at least two other contexts in analyzing voting games⁷⁴ so to have it appear once again is surprising.⁷⁵ The answer also feels about the right size.⁷⁶ Whether the example of Nashville is typical awaits an empirical analysis which, while greatly desirable, is beyond the scope of this paper.

VII. Concluding Remarks

What I have presented in this paper is a positive model of an election that justifies a specific level of at-large representation as maximizing the voting power of the voters. While clearly not dispositive, it does suggest that there are legitimate benefits to be gained by the voters if some appropriate level of at-large representation is provided.

The position that at-large representation is for the public good has a long pedigree. The pitfalls of relying solely on such representation have also been known for a considerable time. Surprisingly there has been no attempt, before now, to try and find the proper balance between them. I hope that this positive analysis will spur others to once again take at-large representation seriously.

⁷¹ The theorem actually states that the optimum number of at-large representatives is the largest integer less than the square root of the total number. The square root of 40 is approximately 6.32 and hence the optimum number is 6.

⁷² These computations were done with the aid of the symbolic system Mathematica. I would be happy to share the scripts with anyone interested. It would be nice to have a theoretical expression for this relative increase of power, but so far a formula has eluded me.

⁷³ Technically it should drop by a factor of $1/\sqrt{2} \approx .707$

⁷⁴ See Felsenthal & Machover at 63.

⁷⁵ And pleasing. I don't know of any unified explanation for the plethora of square roots in this area.

⁷⁶ As in the example of Nashville, where it is off by only 1 from the actual number of at-large representatives.