Introduction to Partial Differential Equations; MATH 4110/7110, Fall 2016; MWF 1:10-14:00, Stevenson 1313

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Office Hours: 1416 Stevenson, MWF 14:00-15:00 or by Appointment

Course Description

This is a basic introduction to Partial Differential Equations. It assumes a working knowledge of (i) Advanced Multivariate Calculus, (ii) Algebra of Matrices, and (iii) a basic theory and solvability methods of Ordinary Differential Equations.

We begin by focusing on the classical Laplace, Heath and Wave equations, and their physical and mathematical motivation. We introduce the notion of Dirichlet, Neumann and Cauchy problem for these equation and solve some of the explicitely. This will be the basis to classifying PDEs, to understanding their structures, and to proceed to more advanced topics such as Galerking methods for non-linear equations, and some non-linear equations arising from fuid-dynamics such as Navier-Stokes equations.

TEXTBOOK AND SUGGESTED BOOKS

No textbook will be required. I will pass along class notes which should be sufficient. The list of books below is meant to complement the lectures and for consultation. They all have been placed in **reserve** in the Sc. Library. While a good portion of topics will be taken from [4], material will be taken from other texts.

SYLLABUS

Basic Modeling Ideas

1. Gauss Green's Theorem
2. Continuity Equation
3. The Heat Equation and the Laplace Equation
4. A Model for the Vibrating String
5. Small Vibrations of a Membrane

Classification of 2nd Order PDE's

1. notion of characteristics
2. Linear PDE's in 2 Variables with Costant Coefficients
3. Classification and Characteristics
4. Classifying Linear PDE's in 2 Variables with Variable Coefficients
5. Classifying Non-Linear PDE's in Several Variables

Vibrating Strings, Surfaces and Volumes

1. The One-Dimensional Wave Equation
2. Properties of Solutions
3. Inhomogeneous Problems
4. A Boundary Value Problems for a Vibrating String
5. Separation of Variables
6. Odd Reflection
7. The Initial Value Problem in N Dimensions
8. Spherical Means
9. The Darboux Formula
10. An Equivalent Formulation of the Cauchy Problem
11. The Cauchy Problem in $\mathbb{R}^3$
12. The Cauchy Problem in $\mathbb{R}^2$
13. The Inhomogeneous Cauchy Problem
14. Separation of variables in 2 and dimensions
15. The Cauchy Problem for Inhomogeneous Surfaces
16. Solutions in Half Space. The Reflection Technique

Coulomb/Newtonian Potentials and the Laplace Equation

1. The Dirichlet and Neumann Problems
2. The Cauchy Problem
3. Well-Posedness and a Counterexample of Hadamard
4. Radial Solutions
5. The Green and Stokes Identities
6. The Green's Function and the Dirichlet Problem for a Ball

7. Sub-Harmonic Functions and the Mean Value Property
8. The Maximum Principle
9. The Harnack Inequality and the Liouville Theorem
10. A Counterexample of Lebesgue for $N = 3$
11. The Poisson Integral fms
12. Non-Homogeneous Problems
13. Separation of variables

The Heat Equation and Heat Conduction Problems

1. The Dirichlet and Neuman Problems
2. The Characteristic Cauchy Problem
3. The Backward Cauchy Problem
4. A Priori Estimates
5. Ill Posed Problems
6. Uniqueness of Solutions to the Cauchy Problem
7. Remarks on the Cauchy Problem
8. About Regularity
9. Instability of the Backward Problem
10. The Inhomogeneous Cauchy Problem
11. Problems in Bounded Domains
12. Separation of Variables
13. The Porous Media Equation
14. The Error Function
15. The Appell Transformation
16. The Heat Kernel by Fourier Transform
17. Rapidly Decreasing Functions
18. The Fourier Transform of the Heat Kernel
19. The Inversion Formula
20. The Blow-Up Phenomenon for Super-Linear Equations

More Advanced Topics (if time permits)

1. Solvability of Elliptic and Parabolic Problems by Galerkin's Method
2. Transmission of Sound Waves
3. The Navier-Stokes System
4. The Euler Equations
5. Isentropic Potential Flows
6. Steady Potential Isentropic Flows

References

[1] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol.I, Interscience, New York, 1953, 1962; (ISBN-10: 0471504475; ISBN-13: 978-0471504474)

[2] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol.II, Wiley-Interscience, New York, 1953, 1962; (ISBN-10: 0471504394; ISBN-13: 978-0471504399)

[3] E. DiBenedetto, *Partial Differential Equations*, Birkhäuser, Boston, 1995; (ISBN: 0-8176-3708-7).

[4] C.L. Evans, *Partial Differential Equations*, Amer. Math. Soc., 1996; (ISBN-10: 0821807722; ISBN-13: 978-0821807729)

[5] P.R. Garabedian, *Partial Differential Equations*, AMS Chelsea (2nd rev Ed. 1998) (ISBN-10: 0821813773; ISBN-13: 987-0821813775).

[6] F. John,*Partial Differential Equations*, Springer–Verlag, New York, 1986 (ISBN 0387–90609–6).

[7] F. John, *Plane Waves and Spherical Means Applied to Partial Differential Equations*, Springer, (1981); (ISBN-10: 0387905650; ISBN-13: 978-0387905655).

[8] O.D. Kellogg, *Foundations of Potential Theory*, Springer Verlag, Berlin 1929, reprinted 1967. (ISBN-10: 0387038159; ISBN-13: 978-0387038155).

[9] O.A. Ladyzenskajia, N.N. Ural'tzeva, *Linear and Quasilinear Elliptic Equations*, Academic Press, London–New York 1968r; (ISBN-10: 0124328504; ISBN-13: 978-0124328501).

[10] O.A. Ladyzenskajia, N.A. Solonnikov, N.N. Ural'tzeva, *Linear and Quasilinear Equations of Parabolic Type*, Transl. Math. Monographs, **23**, AMS Providence R. I., 1968 (ISBN 0–8218–1573–3).

[11] M.M. Lavrentiev, *Some Improperly Posed Problems of Mathematical Physics*, Springer–Verlag, Tracts in Nat. Philosophy, **11**, Berlin–Heidelberg–New York, 1967. (ASIN: B000M3VTGY)

[12] M.H. Protter and H. F. Weinberger, *Maximum Principles in Differential Equations*, Prentice Hall, 1967. (ASIN: B000JEU412).

[13] H. Weinberger, *A First Course in Partial Differential Equations*, Dover, New York 1995; (ISBN-10: 048668640X; ISBN-13: 978-0486686400)