Problem 1 (a)

$y' = t e^{3t} - 2y$

Define $f[t, y] = t e^{3t} - 2y$

Use Euler's method: $y_{n+1} = y_n + h f[t_n, y_n]$

with $y_0 = y[0] = 0$, $t_n = n h$

and $y_n = y[t_n]$

Since $t_n = n h$, $t_1 = 0.5$ and $t_2 = 1$

So, we will seek

$y_1 = y(t_1) = y(0.5)$

and

$y_2 = y(t_2) = y(1)$

```
f[t_, y_] = t ⅇ^(3 t) - 2 y;
y0 = 0;
h = 0.5;
t0 = 0;
t1 = t0 + h = 0    + 0.5 = 0.5
t2 = t1 + h = 0.5 + 0.5 = 1
y1 =
 y0 + h f[t0, y0]
                    = 0 + 0.5 (t0 e^(3 t0) - 2 y0) = 0 + 0.5 (0 - 2*0) = 0
```

0

```
y2 = y1 + h f[t1, y1]
= 0 + 0.5 (t1 e^(3 t1) - 2 y1) = 0 + 0.5 (0.5 e^(3*0.5) - 2 * 0) =
```

1.12042

Problem 1 (b)

$y' = 1 + (t - y)^2$

Define $f[t, y] = 1 + (t - y)^2$

Use Euler's method: $y_{n+1} = y_n + h f[t_n, y_n]$

with $y_0 = y[2] = 1$, $t_n = 2 + n h$, and $y_n = y[t_n]$

Since $t_n = 2 + n h$, $t_1 = 2.5$ and $t_2 = 3$

So, we will seek

$y_1 = y(t_1) = y(2.5)$

and

$y_2 = y(t_2) = y(3)$

```
f[t_, y_] = 1 + (t - y)^2;
y0 = 1;
h = 0.5;
t0 = 2;
t1 = t0 + h =2.5
t2 = t1 + h =3
y1 = y0 + h f[t0, y0]
  = 1 + 0.5 * (1 + (2 - 1)^2) = 1 + 0.5 * 2 =

2.
```

```
y2 = y1 + h f[t1, y1]
= 2 + 0.5 * (1 + (t1 - y1)^2 ) = 2 + 0.5 * (1 + (1.5 - 2)^2) =

2.625
```

Problem 1 (c)

$y' = 1 + y/t$

Define $f[t, y] = 1 + y/t$

Use Euler's method: $y_{n+1} = y_n + h f[t_n, y_n]$

with $y_0 = y[1] = 2$, $t_n = 1 + n h$, and $y_n = y[t_n]$

Since $t_n = 1 + n h$, $t_1 = 1.25$ and $t_2 = 1.5$ and $t_3 = 1.75$ and $t_4 = 2$

So, we will seek

$y_1 = y(t_1) = y(1.25)$

and

$y_2 = y(t_2) = y(1.5)$

and

$y_3 = y(t_3) = y(1.75)$

and

$y_4 = y(t_4) = y(2)$

```
f[t_, y_] = 1 + y / t;
y₀ = 2;
h = 0.25;
t₀ = 1;
t₁ = t₀ + h =1+0.25 = 1.25
t₂ = t₁ + h =1.25+0.25 = 1.5
t₃ = t₂ + h = 1.5 +0.25 = 1.75
t₄ = t₃ + h =1.75 + 0.25 = 2
y₁ = y₀ + h f[t₀, y₀] , so, y₁ = y (t₁) = y (1.25) =

2.75
```

```
y₂ = y₁ + h f[t₁, y₁]

3.55
```

```
y₃ = y₂ + h f[t₂, y₂]

4.39167
```

```
y4 = y3 + h f[t3, y3]
```

5.26905

Problem 1 (d)

$y' = \cos 2t + \sin 3t$
Define $f[t, y] = \cos 2t + \sin 3t$
Use Euler's method: $y_{n+1} = y_n + h f[t_n, y_n]$
with $y_0 = y[0] = 1$, $t_n = n h$, and $y_n = y[t_n]$
Since $t_n = n h$, $t_1 = 0.25$ and $t_2 = 0.5$ and $t_3 = 0.75$ and $t_4 = 1$
So, we will seek
$y_1 = y(t_1) = y(0.25)$
and
$y_2 = y(t_2) = y(0.5)$
and
$y_3 = y(t_3) = y(0.75)$
and
$y_4 = y(t_4) = y(1)$

```
f[t_, y_] = Cos[2 t] + Sin[3 t];
y0 = 1;
h = 0.25;
t0 = 0;
t1 = t0 + h; t2 = t1 + h; t3 = t2 + h; t4 = t3 + h;
y1 = y0 + h f[t0, y0]
```

1.25

```
y2 = y1 + h f[t1, y1]
```

1.63981

```
y3 = y2 + h f[t2, y2]
```

2.02425

```
y4 = y3 + h f[t3, y3]

2.23646
```

Problem 11

Given the initial value problem

$$y' = -y + t + 1,\ 0 \le t \le 5,$$

$y(0) = 1$, with exact solution $y(t) = e^{-t} + t$

a. Approximate y (5) using Euler's method with $h=0.2$, $h=0.1$, and $h=0.05$.

Again, set f[t, y] = −y + t + 1.

For $h = 0.2$, we will need $\frac{b-a}{0.2} = \frac{5-0}{0.2} = 25$ iterations of Euler's method to reach $t_n = 5$;

This is best achieved using some computer program,
for example in MATLAB, using a for loop like :

```
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t = 0;
ycurrent = 1;
h = 0.2;
for  i = 1 : 25
     ynew = ycurrent + h * (-ycurrent + t + 1);
     t = t + h;
     ycurrent = ynew;
end
ycurrent
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The above program calculates the 25 intermediate values of y at t = 0.2,
t = 0.4, t = 0.6, ..., t = 5. At $t = 5$, y (5) = 5.00377789

Running the same code as above,

with $h = 0.1$ requires $\frac{b-a}{0.1} = \frac{5-0}{0.1} = 50$ iterations of Euler's method to reach $t_n = 5$

So, the corresponding MATLAB program is :

```
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t = 0;
ycurrent = 1;
h = 0.1;
n = (5 - 0) / h;
for  i = 1 : n
     ynew = ycurrent + h * (-ycurrent + t + 1);
     t = t + h;
     ycurrent = ynew;
end
ycurrent
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

y = 5.00515377

Lastly, for h=0.05, n=5/h = 100 and y(5) = $y(t_{100}) = 5.00592$, where $t_{100} = 100 * h$

We have not covered the theory pertaining to
part (b) of the problem (won't be required for the test).