Multiple orthogonal polynomials and non-intersecting Brownian paths

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Multiple orthogonal polynomials (MOPs) are an extension of orthogonal polynomials in which the orthogonality is distributed among a number of weights or measures. The MOPs have their origins in Hermite-Padé approximation, but they recently also appeared in stochastic models related to random matrices and non-intersecting random paths.

I will discuss a model of n non-intersecting Brownian bridges with two distinct starting positions and two distinct end positions. The relevant MOPs have orthogonality properties with respect to a rank 1 matrix of Gaussian weight functions and they allow for a description in terms of a 4×4 matrix valued Riemann-Hilbert problem. Depending on the separation between the starting and end positions there are three different regimes in the large n limit. In the regime of large separation the Brownian paths fill two disjoint ellipses, and in the regime of small separation the paths fill a simply connection region bounded by an algebraic curve of degree six. In the regime of critical separation, a surprising connection is found with the Hastings-McLeod solution of the Painlevé II equation.

This is joint work with Steven Delvaux.