MULTIVARIATE STIELTJES CONTINUED FRACTIONS

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For a measure μ on the real line all of whose moments are finite, its moment generating function

$$M^{\mu}(z) = 1 + m_1 z + m_2 z^2 + m_3 z^3 + \dots$$

has a (Stieltjes) continued fractions expansion

$$M^{\mu}(z) = \cfrac{1}{1 - \beta_0 z - \cfrac{\gamma_1 z^2}{1 - \beta_1 z - \cfrac{\gamma_2 z^2}{1 - \beta_2 z - \cfrac{\gamma_3 z^2}{1 - \dots}}}},$$

with coefficients exactly the Jacobi parameters of the measure. Such a measure can equivalently be described as a positive linear functional on polynomials in one variable:

$$\mu : \mathbb{R}[x] \to \mathbb{R}, \qquad \mu[P(x)] = \int_{\mathbb{R}} P(x)\, d\mu(x).$$

In operator theory, one encounters states, which are positive linear functionals φ on polynomials in several (non-commuting) variables

$$\mathbb{R}\langle x_1, x_2, \dots, x_d \rangle.$$

It turns out that their moment generating functions

$$M^{\varphi}(z_1, z_2, \dots, z_d) = 1 + \sum_{i=1}^{d} \varphi[x_i] z_i + \sum_{i,j=1}^{d} \varphi[x_i x_j] z_i z_j + \dots$$

also have continued fraction expansions. I will explain what these look like, and provide a number of examples.