Title. Heisenberg Frame Sets (joint work with Azita Mayeli)

Abstract. Let Λ denote the unitary dual of the Heisenberg group $\mathbb{H}$. We consider a class of subspaces $\mathcal{H}$ of $L^2(\mathbb{H})$ that are invariant under both group translations and automorphic dilations. Each bounded measurable subset I of Λ determines a bandlimited subspace $\mathcal{H}_I \subset \mathcal{H}$ that includes a vector ψ_I whose group Fourier transform is analogous with the characteristic function for I. For the lattice $\Gamma = \Gamma_{\alpha\beta} = \alpha\mathbb{Z} \times \beta\mathbb{Z} \times \mathbb{Z} \subset \mathbb{H}$, application of group translations $T_\gamma, \gamma \in \Gamma$ to a vector ψ reduces on the group Fourier transform side to a one-parameter family of Gabor systems, and this reduction permits a characterization of those subsets I for which $\{T_\gamma \psi_I : \gamma \in \Gamma\}$ is a Parseval frame for $\mathcal{H}_I$. We then introduce automorphic dilations $D_a, a > 0$ and consider the unitary system $\mathcal{U} = \{D_{2^j} T_\gamma : j \in \mathbb{Z}, \gamma \in \Gamma\}$. We prove a necessary condition for general frame vectors for $\mathcal{U}$ that is a precise analogue of [1, Theorem 3.3.1], and we say that I is a *Heisenberg frame set* if ψ_I is a Parseval frame vector for $\mathcal{U}$. Heisenberg frame sets are then characterized in terms of translation and dilation equivalence by an argument that is similar to the proof of [2, Theorem 5.4].

References

[1] I. Daubechies, Ten Lectures on Wavelets, *CBMS-NSF Conference Series in Applied Mathematics* (1992), Capital Series Press

[2] D. Han, D. Larsen, Frames, Bases, and Group Representations, *Mem. Amer. Math. Soc.* **147**, No. 697 (2000)