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1. Know few computations & very few tools:
   a. what are the Steenrod algebras for Bredon cohomology with coeff. in a Mackey functor $\underline{M}$? Orig: $\underline{M}$ a field
   b. What is the Nilpotence theorem?
      - $RO(G)$-graded
2. Algebraic models are often naive: $\mathbb{Z}$-modules, rather than $\underline{A}$-modules
3. Many constructions could possibly benefit from new perspectives.
   induction is coinduction = Wirthmuller isomorphism for finite
4. Rational characteristic classes

$$K(R) \xrightarrow[\text{Bökstedt}]{\text{trace}} THH(R) = S^1 \otimes R$$

1.) THH is genuine S^1-spectrum

2) $K(R) \longrightarrow THH(R)^{S^1}$

Can replace ~~&~~ $THH(R)^{S^1}$ w/ continuous $\mathbb{Q}/\mathbb{Z}$-fixed pts.

$G = \varinjlim F_i \qquad R^{cG} = \varprojlim R^{F_i}$

Localized at p, we instead consider a tower of the fixed pts for $C_{p^n} \in S^1$

Usual method: "Hasse square"

$$\begin{array}{ccccc} E_{hC_{p^n}} & \longrightarrow & E^{C_{p^n}} & \longrightarrow & E^{gC_{p^n}} \\ \| & & \downarrow & & \downarrow \\ E_{hC_{p^n}} & \longrightarrow & E^{hC_{p^n}} & \longrightarrow & E^{tC_{p^n}} \end{array}$$

$\leftarrow$ geometric fixed pts

htpy fixed pts

$a: S^0 \longrightarrow S^V$ Euler class. (for reduced reg. rep.)

$$\begin{array}{ccc} E & \longrightarrow & a^{-1}E \\ \downarrow & & \downarrow \\ E^{\wedge}_{(a)} & \longrightarrow & a^{-1}(E^{\wedge}_{(a)}) \end{array}$$

then take fixed points under C_{p^n} to get previous diagram

Conj: the Euler classes & their "quasi-inverses" are the only nonnilpotent elts in $\pi_\star S^0$. Quasi-inverse: $G = C_2$ $a_\sigma : S^0 \hookrightarrow S^\sigma$

$\bar{\eta} : S^\sigma \longrightarrow S^0$

quasi inv. to idempotent in Burnside ring

THH(R) is cyclotomic.

$THH(R)^{\Phi C_p} \simeq THH(R)$ $\quad S^1/C_p \cong S^1$

Ex: $\mathbb{Z}[x]$, $\quad C_p \otimes \mathbb{Z}[x] = \mathbb{Z}[x, gx, \ldots, g^{p-1}x]$

$\mathbb{Z}[x] \longrightarrow (C_p \otimes \mathbb{Z}[x])^{C_p}$

$f(x) \longmapsto \prod g^i \varphi f(x)$ multiplicative, but not additive

$\mathbb{Z}[x] \longrightarrow (C_p \otimes \mathbb{Z}[x])^{C_p} \longrightarrow \text{"} / \mathrm{Im}(tr)$

$\mathbb{Z}[x] \xrightarrow{\text{surjection}} \mathbb{Z}/p\,[(x \cdot gx \cdots g^{p-1}x)]$

"geometric fixed pt = fixed pts / mod image of transfer"

- need to remember that $C_p \otimes \mathbb{Z}$ should give a Burnside ring $\rightarrow$ this means that "p" is not a transfer + we don't get $\mathbb{Z}/p$.
- that gives a sort of algebraic version of the spectrum level.

$$\begin{array}{ccc} THH(R)^{C_{p^n}} & \xrightarrow{R} & THH(R)^{C_{p^{n-1}}} \\ \downarrow & & \downarrow \\ THH(R)^{hC_{p^n}} & \longrightarrow & THH(R)^{tC_{p^{n-1}}} \end{array}$$

Equivariant	K-theory via trace methods etc
restrictions, transfers } Mackey functor	{ Frobenius, Verschiebung, restriction

$K(R) \longrightarrow \lim\left(TR(R) \overset{1}{\underset{F}{\rightrightarrows}} TR(R)\right)$

$\lim\left(TF(R) \overset{1}{\underset{R}{\rightrightarrows}} TF(R)\right)$