

Speaker: **Larry Brown**

Title: **Non-stable K-theory and extremally rich  $C^*$ -algebras**

Abstract

This is joint work with Gert Pedersen. Our last joint paper has the same title as the talk, and the talk attempts to describe this paper without assuming that the listener is familiar with the previous papers. Extremally rich  $C^*$ -algebras form a class that includes stable rank one  $C^*$ -algebras, purely infinite simple  $C^*$ -algebras,  $W^*$ -algebras, the Toeplitz algebra, and the extended Toeplitz algebra, among others. A simple  $C^*$ -algebra is extremally rich if and only if it is either purely infinite or has stable rank one. The paper seeks to establish non-stable K-theoretic properties for extremally rich  $C^*$ -algebras that are analogous to those known in the purely infinite simple and stable rank one cases, primarily through the work of Cuntz and Rieffel, respectively. We consider the properties weak cancellation,  $K_1$ -surjectivity, good index theory, and  $K_1$ -injectivity. We establish all four properties for isometrically rich  $C^*$ -algebras (isometric richness is a property intermediate between extremal richness and stable rank one, and all prime extremally rich  $C^*$ -algebras are isometrically rich), and for extremally rich  $C^*$ -algebras which are either purely infinite or of real rank zero. ( $K_1$ -injectivity for real rank zero  $C^*$ -algebras was proved by H. Lin, and this result is used in our proof of this case.) We also show that weak cancellation implies the other three properties for extremally rich  $C^*$ -algebras and that the class of extremally rich  $C^*$ -algebras with weak cancellation is closed under extensions.