

An introduction to quantum groups

(Lectures at NCGOA07)

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Summary

Lecture 1: Introduction, historical survey, Hopf algebras, basic examples, algebra of regular functions on $SL(2)$.

Lecture 2: Drinfeld-Jimbo's quantum enveloping algebra $U_q\mathfrak{sl}(2)$, duality with the quantum algebra of functions $SL_q(2)$, tensor categories, category of modules over a Hopf algebra.

Lecture 3: Braid group B_n as the fundamental group of the configuration space of n points, how a solution of the Yang-Baxter equation gives rise to braid group representations, braiding in a tensor category, braided Hopf algebras, universal R -matrix.

Lecture 4: Construction of the categorical double, crossed bimodules, application to the construction of a universal R -matrix in the case of a group algebra.

Lecture 5: Infinitesimal braidings, the Knizhnik-Zamolodchikov flat connection on the configuration space of n points, braid group representations from its monodromy, Kohno-Drinfeld's theorem, Kontsevich's integral.

References

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