Boundedness of noncommutative martingale transforms and an approximation result for hyperfinite martingales

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Inspired by the classical theory, noncommutative probability is motivated by quantum physics. Gilles Pisier and Quanhua Xu proved that, under certain conditions on the filtration, a noncommutative martingale can be transferred to a commutative vector-valued martingale for which classical theory applies. We will show that in the setting of a hyperfinite von Neumann algebra, an L_p-martingale can be approximated in the p-norm ($1 < p < \infty$) by martingales with respect to finite dimensional filtrations, for which the argument of Pisier and Xu applies. The proof relies on a perturbation argument using approximate matrix units techniques. We use this result to study the operator space UMD property, introduced in this context by Pisier, and establish connections with the Banach space property.