MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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C^*-ALGEBREN

01.02.-07.02.1998

Die Tagung fand unter Leitung von Dietmar Bisch (Santa Barbara), Eberhard Kirchberg (Berlin) und Georges Skandalis (Paris) statt.

Es haben wie schon bei früheren Tagungen auch in diesem Jahr wieder viele hochkarätige Mathematiker aus mehr als einem Dutzend verschiedener Länder an der C^*-Algebren Tagung in Oberwolfach teilgenommen, und es war deshalb nicht schwer ein hervorragendes Tagungsprogramm zusammenzustellen.

In den Vorträgen wurden die neusten Resultate aus den unterschiedlichen Teilgebieten der Theorie der C^*-Algebren, wie zum Beispiel der von Vaughan Jones begründeten Theorie der Unterfaktoren, der freien Wahrscheinlichkeitstheorie, KK-Theorie, Klassifizierung von C^*-Algebren und der algebraischen Quantenfeldtheorie präsentiert. So wurden zum Beispiel Konstruktionen von neuen C^*-Algebren mit unterschiedlichen Invarianten erläutert, Anwendungen von nichtkommutativen Methoden zur Topologie und Geometrie (z.B. Novikov Vermutung, Baum-Connes Vermutung) wurden präsentiert und ein Konzept analog zur Eigenschaft T von Kazhdan für Standardinvarianten von Unterfaktoren wurde dargestellt. Weiterhin wurden kombinatorische und/oder darstellungstheoretische Eigenschaften von Operatoralgebren und neue Resultate über Spektraleigenschaften und invariante Unterräume der circular elements von Voiculescu wurden diskutiert.

Wir haben oben nur einige wenige der reichhaltigen Strukturen, die in natürlicher Weise in der Theorie der C^*-Algebren auftauchen, aufgeführt. Im Tagungsprogramm waren im wesentlichen alle Richtungen der augenblicklichen Forschung enthalten und viele der Vorträge wurden von jungen Forschern gehalten.

Vortragsauszüge

Joachim Cuntz

A general construction of bivariant K-theories

In [Cu] a general mechanism was developed that allows the construction of bivariant K-theories on various categories of topological algebras. If $\mathfrak{A}$ is such a category with suitable notions of a tensor product, of homotopy, of matrix stability and with a class of extensions $\mathfrak{E}$ containing suspension, dual suspension and universal extensions, then there is a functor kk from $\mathfrak{A}$ into an additive category $kk(\mathfrak{A})$ (whose objects are the objects of $\mathfrak{A}$). The functor kk is homotopy invariant, matrix stable and exact on extensions in $\mathfrak{E}$. Moreover, kk is the universal functor with these properties.

Specializing to the category of C*-algebras (or, more generally C*-algebras with action of a locally compact group G) we obtain Kasparov's KK-fucntor or the E-functor of Connes-Higson (or intermediate bivariant theories), depending on whether we choose for $\mathfrak{E}$ the class of all extensions with completely positive lifting or of all extensions (or intermediate choices), respectively.

This leads to some new connections between KK and E. For instance, both theories are connected by a six-term exact sequence, using mapping cones and a relative theory. Also, we obtain a new criterion on A so that $E(A, B) \cong KK(A, B)$ for all B.

[Cu] Documenta Math. 1997.

Jesper Villadsen

The stable rank of simple C-algebras*

In this talk I will describe the construction of a simple, unital and approximately homogeneous C*-algebra in which the invertible elements do not form a dense subset. So the algebra does not have stable rank one. A variation of the construction gives simple C*-algebras of stable rank any given natural number.

Jean-Louis Tu

The Baum-Connes conjecture for groupoids with Haar systems and applications to foliations

We extend the definition of the Baum-Connes map for groups to locally compact groupoids endowed with a Haar system, by using the tools of equivariant KK-theory, which is a convenient framework to establish "dual Dirac-Dirac"-type constructions. We then explain how this can be applied to foliations. In particular, the Baum-Connes map is injective for hyperbolic foliations with compact base and whose holonomy groupoid is Hausdorff, and that implies the Novikov conjecture on higher signatures for such foliations. The theory can also be used to study amenable groupoids.

Marius Dadarlat

Quasidiagonal C-algebras (two applications of residually finite C*-algebras)*

1. It is shown that the class of simple, unital, quasidiagonal C*-algebras, of real rank zero and stable rank one, with unique trace, contains nonexact C*-algebras and exact nonnuclear C*-algebras. That solves a problem of Popa and answers questions of Hadwin and Rosenberg.

2. Any quasidiagonal representation of a separable exact C*-algebra, whose image does not contain any nontrivial compact operators, is a point-norm limit of completely positive contractions whose images generate finite dimensional C*-algebras. Let G be a discrete, countable, residually finite dimensional group. If G is amenable, then the left regular representation is a point-norm limit of finite dimensional representations. On the other hand, it was shown by Voiculescu, that if G has Property T, then C*(G) has quasidiagonal representations which are not approximable as above.

Simon Wassermann

Exact groups and continuous bundles of C-algebras*

Let G be a locally compact group with a continuous action $\alpha : G \to \mathrm{Aut}(A)$ on a C*-algebra A. If J is an $\alpha(G)$-invariant closed two-sided ideal of A, with embedding map $\iota : J \to A$ and quotient map $q : A \to A/J$, the corresponding *-homomorphisms of the reduced crossed products of these algebras by G, $\iota_{\alpha,r} : J \rtimes_{\alpha|J,r} G \to A \rtimes_{\alpha,r} G$ and $q_{\alpha,r} : A \rtimes_{\alpha,r} G \to (A/J) \rtimes_{\dot{\alpha},r} G$, are injective and surjective, respectively, but, while Im $\iota_{\alpha,r} \subseteq \ker q_{\alpha,r}$, it is not obvious in general that this inclusion is an equality. The group G is said to be *exact* if the inclusion is always an equality, i.e. if, for arbitrary A and α-invariant ideal J of A, the sequence

$$0 \to J \rtimes_{\alpha|J,r} G \to A \rtimes_{\alpha,r} G \to (A/J) \rtimes_{\dot{\alpha},r} G \to 0$$

is exact. Equivalently, if $\mathcal{C}_G^*$ is the category whose objects are the pairs (A, α) consisting of C*-algberas A and continuous actions α of G on A, and whose maps are the equivariant *-homomorphisms, then G is exact if and only if the functor $(A, \alpha) \to A \rtimes_{\alpha,r} G$ from $\mathcal{C}_G^*$ to the category of C*-algebras is short-exact.

It follows easily from this definition that if G is exact, then the reduced C*-algebra $C_r^*(G)$ is an exact C*-algebra, and the converse holds if G is discrete. We show that exactness if preserved on passing to closed subgroups, that the extension of an exact group by an exact group is exact, and that a group containing a closed exact subgroup with finite covolume is itself exact. One of the main tools used in the proofs is a generalisation of the imprimitivity theorems of P. Green and M. Rieffel. These results imply that a wide variety of known groups, including in particular all connected locally compact groups, free groups and matrix Lie groups, is exact.

We also give a characterization of exact groups which involves actions on continuous bundles of C*-algbras. A groups G is exact if and only if for any continuous C*-bundle $\mathcal{A}$ over a locally compact Hausdorff space X with fibre A_x at $x \in X$ and fibrewise actions $\alpha_x : G \to \mathrm{Aut}(A_x)$ which define a continuous action α of G on the bundle algebra A, the crossed-product bundle

$$\mathcal{A} \rtimes_{\alpha,r} G = (A \rtimes_{\alpha,r} G, X, \pi_{\alpha,r,x} : A \rtimes_{\alpha,r} G \to A_x \rtimes_{\alpha_x,r} G)$$

is continuous. We show that to determine the exactness of a given group G, it suffices to check the continuity of the reduced crossed product by G of a particular continuous bundle, depending canonically on G. (Joint work with Eberhard Kirchberg).

Mikael Rørdam

Stability and properly infiniteness are not stable properties

A projection in a C*-algebra is said to be properly infinite if it has two mutually orthogonal subprojections each of which is equivalent to the given projection. A unital C*-algebra is properly infinite if the unit is a properly infinite projection. Equivalently, a unital C*-algebra A is properly infinite if there is a unital embedding of O_∞ into A.

One significance of properly infinite C*-algebras is the fact that a unital C*-algebra A does not admit a quasi-trace (or an exact C*-algebra A does not admit a trace) if and only if $M_n(A)$ is properly infinite for some n. This raises the question if $M_n(A)$ is properly infinite implies that A is properly infinite (in which case the theorem above would read: A admits a (quasi-)trace if and only if A is not properly infinite). Also, an affimative answer to this question would imply that no simple C*-algebra could contain at the same time an infinite and a (non-zero) finite projection.

Using techniques of Jesper Villadsen, we show that there exists a (simple, nuclear, stable rank one, separable) C*-algebra A so that $M_2(A)$ is stable while A is not stable. From this we give an example of a (non-simple) unital C*-algebra B such that $M_2(B)$ is properly infinite while B is not properly infinite, thus answering the question raised above in the negative. Although these examples do not answer the question, if simple C*-algebras can admit simultaneously infinite and finite projections, they may indicate that the answer to that question might be also be "no".

Florin Boca

Projections in A_θ and Theta

If $\theta \in (0,1)$, the rotation C^*-algebra A_θ associated with the rotation by angle $2\pi\theta$ of the unit circle, is the universal C^*-algebra generated by two unitaries u and v subject to the commutation relation $uv = e^{2\pi i\theta} vu$. Consider also the order four automorphism σ of A_θ defined by $\sigma(u) = v$, $\sigma(v) = u^{-1}$ and the fixed point algebra $A_\theta^\sigma = \{x \in A_\theta\,;\, \sigma(x) = x\}$.

We develop a method for constructing projections in A_θ^σ of trace θ, $q\theta - p$ if $0 < q\theta - p < (2q)^{-1}$, $p = p_0^2 \bmod q$ for some integer p_0 and $p - q\theta$ if $0 < p - q\theta < (2q)^{-1}$, $p = -p_1^2 \bmod q$ for some integer p_1, using Rieffel's formalism and Jacobi's theta functions.

These projections are being used to obtain lower bounds for the norm of the almost Mathieu operator $h_\theta = u + u^* + v + v^*$. If $\theta = q^{-1}$ for some integer $q \geq 2$, they can be expressed in an explicit way through some operator-valued theta functions.

Joachim Zacharias

Some new results about continuous analogues of Cuntz algebras

Arveson has defined the notion of a product system and associates a C*-algebra to it which may be considered as a continuous analogue of the Cuntz algebras. Using recent dilation techniques from the theory of completely positive semigroups we dilate any such algebra to a crossed product and show simplicity by computing the Connes spectrum. In particular all the continuous Cuntz algebras are simple. They are also KK-contractible. Next we give a description of unital endomorphisms (i.e. those whose image contains an approximate unit) and their action on the dual in terms of cocycles. We look at the continuous analogue of the quasifree automorphisms. We also look at certain weights connected with them.

Edward Effros

Integral mappings and the principle of local reflexivity for non-commutative L^1-spaces

The second dual V^{**} of a Banach space V has the same local structure as V. By this we mean that if F is a finite dimensional subspace of V^{**}, and $\varepsilon > 0$, then there is a subspace E of V with Banach-Mazur distance $d(E,F) < \varepsilon$. This result is a consequence of the principle of local reflexivity, which holds for all Banach spaces. The analogous completely bounded version of this result is false for C^*-algebras A. We have, for example, that $B(H) = K(H)^{**}$ must contain a finite dimensional subspace F and an $\varepsilon > 0$ such that for any subspace $E \subseteq K(H)$, the Pisier-Banach-Mazur distance $d_{cb}(F,E) \geq 1+\varepsilon$. This is a simple consequence of the fact that $K(H)$ is exact in the sense of Kirchberg, whereas $B(H)$ does not have that property. It is thus quite remarkable that the predual of *any* von Neumann algebra satisfies the full analogue of the principle of local reflexivity. This is proved by using the Kaplansky density theorem as in Junge's Habillitationschrift, together with a careful analysis of the notion of integrality for mappings of operator spaces. In principle, this result should provide new invariants for von Neumann algebras that are preserved when one takes second duals, and it might also provide a first step in understanding the local structure of the preduals of von Neumann algebras. (Joint work with M. Junge and Z.-J. Ruan).

Roberto Longo

A quantum index theorem, JLO cocycle and Jones index

Let A be a unital C*-algebra with trivial center and $\tau \subset \mathrm{End}(A)$ a tensor category occurring in Quantum Field Theory, where each endomorphism $\rho \in Obj(\tau)$ has a conjugate. Let α be a one parameter automorphism group of A and $u(\rho, t)$ a two variable cocycle, in particular $u(\rho, t)$ is a unitary of A and

$$\mathrm{Ad}\, u(\rho, t) \circ \alpha_t \circ \rho \circ \alpha_{-t} = \rho.$$

If ω is a faithful KMS state, then $u(\rho, t)$ is essentially holomorphic and setting

$$d_{top}(\rho) = \sqrt{d_{hol}(u) d_{hol}(u^*)}$$

where $u = u(\rho, t)$, $d_{hol}(u) = \text{anal. cont.}_{t \to i} \omega(u(t))$, we have $d_{an}(\rho) = d_{top}(\rho)$, where $d_{an}(\rho)$ is the square root of the Jones index of ρ.

In a supersymmetric QFT, $d_{top}(\rho)$ coincides with the index associated with the JLO cocycle associated in a natural way with a (graded) KMS functional ω_ρ for $\mathrm{Ad}\,(u(\rho, t))\alpha_t$.

On a curved, globally hyperbolic space time M, with bifurcate Killing horizon,

$$d_{an}(\rho) = d_{top}(\rho) = e^{-\beta k F(\omega | \omega_\rho)}$$

where $k = k(M)$ is the surface gravity and $F(\omega|\omega_\rho)$ is the increment of the free energy at Hawking temperature β^{-1} (no supersymmetry in this case).

Claire Anantharaman-Delaroche

Amenable groupoids

After Zimmer's introduction and study of amenability for ergodic group actions or equivalence relations, a major breakthrough has been the Connes-Feldman-Weiss characterization of amenable countable measured equivalence relations by hyperfinitness. Since then, Zimmer's theory has been applied to various contexts, but its extension to arbitrary measured or topological groupoids remained to be completed, and the need for clarification was recently pointed out by several authors. This is the subject of our work.

This talk will be a survey on old and new results on the various notions of amenability, their relations and applications to operator algebras. (Joint work with Jean Renault).

Etienne Blanchard

Finite dimensional Hopf C^*-algebras

It has been proved by D. Ştefan through cohomological arguments that there exists only a finite number of Hopf C^*-algebras of a given finite dimension N (in fact, one can find a rough estimate of this number thanks to the multiplicative unitaries framework). In a joint work with S. Baaj and G. Skandalis, we study these finite dimensional Hopf C^*-algebras and their sub-objects from the point of view of multiplicative unitaries.

Given a (non zero) Hilbert space $\mathcal{H}$ of finite dimension N and an irreducible multiplicative unitary $V \in \mathcal{L}(\mathcal{H} \otimes \mathcal{H})$, define a pre-subgroup of V to be a unit vector $f \in \mathcal{H}$ satisfying the relation $V(f \otimes f) = f \otimes f$. In the case of the multiplicative unitary associated to a finite group G ($\mathcal{H} = l^2(G)$), such a pre-subgroup is (proportional to) the characteristic function of a subgroup of G. The space of pre-subgroups admits a natural structure of lattice, so that one can define the subgroup generated by a set, the normaliser of a pre-subgroup, *etc.* Furthermore, if f, g are two subgroups of V, then $|\langle f, g\rangle|^{-2}$ is an integer, whence an explicit upper bound of the number of pre-subgroups of V (*cf.* Izumi and *co*). Notice that there is a natural correspondence between the pre-subgroups of a multiplicative unitary and the coideals of a finite dimensional Hopf C^*-algebra or the intermediate subfactors of an irreducible depth 2 inclusion of finite depth (*cf.* Izumi, Longo, Popa).

We also provide a generalisation of the « bicrossed products» studied by Takeuchi and Majid. Given two pre-subgroups f and g which are the most possible remote, *i.e.* such that $|\langle f, g\rangle|^{-2} = N$, we construct a new multiplicative unitary $W \in \mathcal{L}(\mathcal{H} \otimes \mathcal{H})$ such that, in the case of two subgroups G_1, G_2 of a finite group G such that $G = G_1G_2$, $G_1 \cap G_2 = \{e\}$, the associated Hopf C^*-algebras are the crossed products $S_W = C(G/G_2) \rtimes G_2$ and $\widehat{S}_W = C(G_1 \backslash G) \rtimes G_1$.

Ola Bratteli

Compactly supported wavelets and representations of the Cuntz relations

We study the harmonic analysis of the quadrature mirror filters coming from multiresolution wavelet analysis of compactly supported wavelets. It is known that those of these wavelets that come from third order polynomials are parametrized by the circle, and we compute that the corresponding filters generate irreducible mutually disjoint representations of of the Cuntz algebra $\mathcal{O}_{\in}$ except at two points on the circle. One of the two exceptional points corresponds to the Haar wavelet and the other is the unique point on the circle where the father function defines a tight frame which is not an orthonormal basis. At these two points the representation decomposes into two and three mutually disjoint irreducible representations, respectively, and the two representations at the Haar point are each unitarily equivalent to one of the three representations at the other singular point. (Joint work with Dai Evans and Palle Jorgensen).

Pierre de la Harpe

Groups with reduced C-algebras of stable rank one*

For a group Γ, let $C^*(\Gamma)$ denote the corresponding reduced C*-algebra. Recently, U. Haagerup, M. Rørdam and the first author have shown that, if Γ is a non-abelian free group, then $C^*(\Gamma)$ has stable rank one.

We extend this result to other groups, including groups which are non-elementary hyperbolic in the sense of Gromov and torsion free. We discuss also the open problem to decide whether these group algebras have real rank one. (Joint work with Ken Dykema).

Adrian Ocneanu

Modular invariants, Kleinian invariants, platonic solids and subfactors

The minimal model classification of Itzykson, Capelli, Zuber (ICZ) gives matrices of intertwiners for the representation of $SL(2, \mathbb{Z})$ introduced by Hurwitz and known as the Verlinde representation in physics. We show how to interpret the entries of these matrices as

1) invariants of $SU(2)_N$ (in the same way in which invariants of the Kleinian subgroups G of $SU(2)$ determine the representation theory of G) and

2) character of fusion algebras of these subgroups.

This provides an illustration for the following rigidity results:

a) There are only countably many finite depth subfactors of the hyperfinite II_1 factor (equivalently: There are only finitely many rational topological quantum field theories in 3 dimensions, $TQFT_3$, with a given fusion algebras).

b) The maximal atlas of bimodules for a finite depth subfactor is finite (equivalently: There are only finitely many types of punctures - or vertices - for a given rational $TQFT_3$) and

c) There are only finitely many possible braidings in a rational $TQFT_3$.

We show that using Kleinian type invariants one can construct explicitly all the irreducilbe objects (bimodules) in a theory, all the intertwiners and all the $6j$ symbols.

Yasuyuki Kawahigashi

Quantum Galois correspondence for strongly amenable subfactors

We extend quantum Galois correspondence between generalized intermediate subfactors and subequivalent paragroups to strongly amenable subfactors. Our previous work on the finite depth case used Sato's construction which heavily relies on Ocneanu's compactness argument and estimates of global indices. These two tools both fail for the infinite depth case, so we first generalize Sato's construction to the infinite depth case by working on amenable fusion rule algebras in the sense of Hiai-Izumi and using "standard invariants" for intermediate subfactors.

As an application of this generalization, we clarify a relation between our subequivalent paragroups and sublattices of a standard λ- lattice in the sense of Popa. That is, suppose that a standard λ-lattice Λ_1 is a strongly amenable sublattice of another one Λ_2 with finite index. (The index of a sublattice is naturally defined. The assumption automatically holds if Λ_1 has a finite depth.) Then the paragroup of Λ_2 is subequivalent to that of Λ_1. Note that the inclusion order is reversed. For this purpose, we generalize open string bimodules of Ocneanu and Asaeda-Haagerup to infinite graphs.

Hans-Werner Wiesbrock

Modular Theory in Quantum Field Theory

I will give a review on modular inclusions and intersections of von Neumann algebras and how they naturally occur in the algebraic approach to quantum field theory. A modular inclusion is defined as follows. Given an inclusion of von Neumann algebras together with a common cyclic and separating vector. If the associated modular group of the larger one maps the subalgebra into itself for positive or negative parameter, we call it a modular inclusion. (This notion was generalized to weights by Araki/Zsido.) They show up an interesting symmetry structure and are the building blocks for modular intersections. Together these strucures enabled one to describe a quantum field theory by an appropriate finite set of algebras with a common cyclic and separating vector.

Sorin Popa

Symmetric enveloping algebra, amenability and property T for subfactors

Let $N \subset M$ be an extremal inclusion of II_1 factors with finite Jones index, $[M : N] < \infty$. In short, its *symmetric enveloping algebra* is the unique (up to isomorphism) II_1 factor S which is generated by mutually commuting copies of M and M^{op}, satisfying $M' \cap S = M^{\mathrm{op}}$, $M^{\mathrm{op}\prime} \cap S = M$, and by a projection e_N whiich implements both the trace preserving conditional expectations of M onto N and of M^{op} onto N^{op}.

S has several interesting decomposition properties. Thus, it can be written as some type of cross-product over the subalgebra $M \vee M^{\mathrm{op}} \simeq M \otimes M^{\mathrm{op}}$. Also, if M is hyperfinite then S follows *thin*, i.e. there exist hyperfinite subfactors R_1, $R_2 \subset S$ such that $S = \overline{\mathrm{span}}\, R_1 R_2$.

Moreover, we prove that S is itself hyperfinite if and only if the graph $\Gamma_{N,M}$ of $N \subset M$ is amenable (besides M being hyperfinite). Also, we present a number of other results relating the amenability of $\Gamma_{N,M}$ with properties of S.

Finally, we introduce a notion of property T for the standard invariant associated to subfactors, i.e. the group-like object $\mathfrak{G}_{N,M}$, by requiring the symmetric enveloping algebra S of $N \subset M$ to have the property T relative to its subalgebra $M \vee M^{\mathrm{op}}$. This definition relies on a theorem, showing that the above does not in fact depend on the subfactor taken, but just on $\mathfrak{G} = \mathfrak{G}_{N,M}$. We present the proof of this theorem and some resulting consequences for the property T.

Dietmar Bisch

Examples of subfactors with property T standard invariant

Popa introduced recently a notion of property T for the standard invariant of a subfactor. We give a construction of a large class of subfactors with property T. These subfactors are of the form $M^H \subset M \rtimes K$, where H and K are finite groups with a properly outer action on the hyperfinite II_1 factor M. We prove that these subfactors have property T in the sense of Popa if and only if the group G generated by H and K in the outer automorphism group of M has the property T of Kazhdan. Furthermore we construct explicitly such subfactors which are irreducible, have infinite depth and small Jones index. (Joint work with Sorin Popa).

Teodor Banica

Representations of compact quantum groups and subfactors

If G is a compact quantum group and π is a finite dimensional representation of G then the algebras of the form

$$End(\ldots \otimes \pi \otimes \hat{\pi} \otimes \pi \otimes \hat{\pi} \otimes \ldots)$$

form a lattice, say $L(\pi)$, which satisfies Popa's axioms for higher relative commutamts of subfactors. We characterise the class of lattices arising in this way - these are the ones which satisfy Popa's four axioms, plus a fifth axiom which says that the lattice can be "represented" on a finite dimensional Hilbert space. This is proved via a universal construction. The fifth axiom forces the index to be greater that 4, and by using a Kesten type result we find also that in the amenable case, this extra axiom forces the index to be the square of an integer.

The question "when (G, π) and (G', π') give the same Popa system ?" is also discussed (this turns to be related to free products).

The subfactors coming from vertex models satisfy the above axioms (this follows from Jones' description of the commutamts using diagrams), and we present in this case a quite explicit construction for the compact quantum group (which is different from the universal one constructed in the general case).

George Elliott

The spectrum of a C^*-algebra

As a simplified context in which to study the classification problem for separable amenable C*-algebras, in the non-simple case, it seems natural to consider the class of such C*-algebras absorbing both the C*-algebra K of compact operators and the Cuntz algebra O_2 as tensor product factors – in other words, those which are stable and also what might be called O_2-stable.

For this class of C*-algebras the only obvious invariant is the lattice of closed two-sided ideals – equivalently, the spectrum.

(While another candidate might be the semigroup of Murray-von Neumann equivalence classes of projections, it turns out that this is derivative, being naturally isomorphic to the semigroup of compact open subsets of the spectrum, with union as the semigroup operation; this is a consequence of the (proof of the) Blackadar- Cuntz theorem on existence of projections.)

A survey is given of the already interesting body of results concerning this class of algebras. In particular, the existence of a unique (non-zero) simple C*-algebra in this class follows from the work of Kirchberg. The algebras in this class which have totally ordered ideal lattice, and are assumed to arise as inductive limits of tensor products of a commutative C*-algebra with $O_2 \otimes K$, were classified by Jakob Mortensen in his Odense Ph.D. thesis.

Stanislaw Woronowicz

Quantum "$ax+b$" group on the Hilbert space level

On the Hopf *-algebra level the quantum "$ax+b$" group is an object with no problem. The polynomial algebra is generated by two selfadjoint elements a, b satisfying the relation $ab = qba$, where q is a complex number of modulus 1. Comultiplication is given by the formulae:

$$\Delta(a) = a \otimes a$$

$$\Delta(b) = a \otimes b + b \otimes I.$$

It turns out that on the Hilbert space level the operator $\Delta(b)$ is symmetric but not selfadjoint. This means that the comultiplication on the corresponding C*-algebra does not exist. We shall show the way out of this difficulty. If the time permits, the confrontation with the quantum SU(1,1) group will be discussed.

Uffe Haagerup

Spectral analysis of Voiculescu's circular element and other non-normal operators in free group factors

In 1983 L.G.Brown used the Kadison-Fuglede determinant to define a "spectral distribution measure" for any (not necessarily normal) operator in a factor of type II_1. We show that for Voiculescu's circular element, this distribution measure becomes the uniform distribution on the unit disk in the complex plane. Moreover we find an explicite formula for the spectral distribution measure of any operator of the form $T = UH$, where (U, H) is a *-free pair of operators, U is Haar-unitary and H is positive, in terms of the S-transform of the distribution of T^*T. (The class of operators considered here are those for which (T, T^*) is an R-diagonal pair in the sense of Nica and Speicher). We also compute Browns spectral distribution measure for some operators outside this class, namely operators of the form $a_1X_1 + a_2X_2$, where (X_1, X_2) is a semicircular system, and a_1, a_2 are arbtrary complex numbers. Part of the results mentioned above are obtained in collaboration with Flemming Larsen (Odense).

Ken Dykema

Invariant subspaces of the circular element

The circular element arises naturally in Voiculescu's free probability theory. It is $Y = X_1+iX_2$, where X_1 and X_2 are free semicircular elements of the same normalization. The von Neumann algebra generated by the circular element is $L(F_2)$. One of Voiculescu's important results with implications for the free group factors is that a matrix whose entries are free circular elements is in turn a circular element.

We show that also a certain upper–triangular matrix is a circular element. We use this to find invariant subspaces for the circular element, namely for every $0 < t < 1$ we find a projection p in $L(F_2)$, having trace t, such that $Yp = pYp$. Our proof uses random matrices.

Roland Speicher

Free cumulants and free entropy

It has turned out that many properties of freeness can be described very nicely and effectively in terms of so-called free cumulants. In the first part of my talk I will survey these objects and some of their applications. In the second part I will switch to free entropy. This is a more geometrical kind of object and has no direct description in terms of free cumulants. Nevertheless, via Voiculescu's approach to free information and free entropy in terms of a non-commutative Hilbert transform, there exists a connection. I will show how this enables one to calculate the free entropy of special two-dimensional distributions, namely of so-called R-diagonal pairs.

Florin Radulescu

A generation property for von Neumann algebras associated to free groups

The following result is proved:

Theorem. *Let B be the von Neumann algebra $L(PSL(2,\mathbb{Z})) \otimes B(H)$. Then there exists a bounded, subnormal operator Z, in B such that B is the weak closure of Span $\{(Z^m)^* Z^n | m, n = 0, 1, ...\}$.*

In a specific model Z is the Toeplitz operator, on a suitable Hilbert space of analytic functions, with (analytic symbol) j, where j is the modular invarint function. The Hilbert space is taken so that multiplication by j is bounded.

There exist obstructions for the Toeplitz operator with symbol $\overline{j}$ to be extended to a closable operator on the functions on $\mathbb{H}$. This also gives an example of a non zero, unbounded Hankel operator: $PM_{\overline{j}}(I-P)$ with antianalytic symbol (it is well known that bounded, antianalytic symbols give rise to the null operator).

Corollary. *Let G be a fuchsian group of finite covolume, and $j = \frac{a}{b}$ an analytic, G invariant function on $\mathbb{H}$ as above. Let P_t be the projection from $L^2(\mathbb{H}, (\mathrm{Im}\ z)^{t-2} \mathrm{d}\overline{z}\mathrm{d}z)$ onto $H_t = H^2(\mathbb{H}, (\mathrm{Im}\ z)^{t-2}\mathrm{d}\overline{z}\mathrm{d}z)$. Let $M_{\overline{j}}$ be the multiplication operator with $\overline{j}$ which is defined on a dense subset of the Hilbert space $L^2(\mathbb{H}, (\mathrm{Im}\ z)^{t-2}\mathrm{d}\overline{z}\mathrm{d}z)$. Then $P_t M_{\overline{j}}(1-P_t)$ is nonzero (in particular has nonzero domain).*

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