ABSTRACTS

Multivariate Stochastic Korovkin Theory Given Quantitatively

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We introduce and study very general multivariate stochastic positive linear operators induced by general multivariate positive linear operators that are acting on multivariate continuous functions. These are acting on the space of real differentiable multivariate time stochastic processes. Under some very mild, general and natural assumptions on the stochastic processes we produce related multidimensional stochastic Shisha Mond type inequalities of L_q type, $1 \leq q < \infty$, and corresponding multidimensional stochastic Korovkin type theorems. These are regarding the stochastic q-mean convergence of a sequence of multivariate stochastic positive linear operators to the stochastic unit operator for various cases. All convergences are produced with rates and are given via the stochastic inequalities involving the maximum of the multivariate stochastic moduli of continuity of the nth order partial derivatives of the engaged stochastic process, $n \geq 0$. The astonishing fact here is that basic real Korovkin test functions assumptions are enough for the conclusions of our multidimensional stochastic Korovkin theory. We give an application.

A Limit Theorem for Szego Polynomials with respect to Convolution

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Two recently-proposed methods for estimating the m frequencies of a trigonometric signal using Szego polynomials of fixed degree $k > m$ consist of multiplying the moments of the n-truncated periodogram by the moments of the Poisson kernel and the wrapped Gaussian, respectively, in an effort to address the non-convergence of the polynomials as n approaches infinity. These methods are seen to be equivalent to convolution of point masses with approximate identities, suggesting a general method. We characterize the limit polynomial for the case when the approximate identity is the Fejer kernel, extending recent results of the author for the case of the Poisson kernel. Moreover, the limit is seen to be the same as in the former case.

Trivariate Spline Approximations of Divergence-Free Vector Fields

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We discuss the approximation properties of divergence-free vector fields by using trivariate spline vectors which are also divergence-free. We pay special attention to the approximation constants and show that they depend only on the smallest solid angle in the underlying tetrahedral partition and the nature of the boundary of the domain. The estimates are given in the max-norm and L^p norm.

Asymptotically optimal choice of knots for spline interpolation and applications to numerical integration

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In this talk we will discuss some questions of adaptive approximation by various classes of splines (linear, multilinear, biquadratic). In particular, we will study the asymptotic behavior of the optimal error of weighted approximation in different norms by interpolating splines from these classes.

One of the standard applications of adaptive choice of knots is to design cubature formulae which are optimal on the class of functions we consider and exact on a certain subset of it.

An example of results obtained is a cubature formula for computing a weighted integral with positive weight $\Omega \in C(D)$ of an arbitrary function $f \in C^2(D)$, $D = [0,1]^2$, such that its Hessian $H(f;x,y)$ is positive and bounded away from zero. We shall present estimate for the error as well. To introduce the formula for the error we shall construct an asymptotically optimal triangulation $\triangle_N^*$ where N is the number of vertices in the triangulation. If $E(f,\triangle_N^*)$ denotes the error of the cubature formula then we show that

$$E(f,\triangle_N^*) \leq \|\Omega\|_p^{1/p} \frac{C_p^+}{2N} \left(\int_D H(f;x,y)^{\frac{p}{2(p+1)}} \Omega(x,y)^{\frac{1}{p+1}} dxdy \right)^{\frac{p+1}{p}}, \quad p \in [1,\infty],$$

where C_p^+ is a certain constant which will be characterized. This formula is exact on the piecewise linear functions corresponding to the partition $\triangle_N^*$.

Algebraic Spline Molecular Models from Electron Microscopy

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In this paper I shall describe an intertwined signicant set of image processing, computational geometry, computational topology and algebraic spline approximation algorithms to reconstruct molecular structural models of molecules from Electron Microscopy (EM) Imaging. I shall also allude to the increased use of these molecular models in approximating binding energetics, a necessary step towards computational drug discovery.

Minimal energy splines for hole filling on planar and spherical domains.

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We conduct numerical investigation on how degree, smoothness, density of the data affects minimal energy splines used to fill holes. We experiment with both planar and spherical domains, convex and nonconvex.

Uniform approximation by weighted polynomials

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Let $w(x)$ be a continuous non-negative weight on the real line which decays faster than $1/|x|$. The topic of uniform approximation by weighted polynomials $w(x)^n P_n(x)$ was introduced by Saff. He conjectured that $w(x)^n P_n(x)$ $(n = 0, 1, 2, ...)$ are dense on the support of the equlibrium measure, assuming $\log w(x)$ is concave. This was proved by Totik. In the talk we give other sufficient conditions on $w(x)$ which imply denseness.

On Durrmeyer Type Operators and Quasi-Interpolants

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In the talk we present our recent results on the Durrmeyer type operators and quasi-interpolants. Our consideration includes the Bernstein-Durrmeyer operator in the L_p-spaces with Jacobi weights on the d-dimensional simplex, the Baskakov-Durrmeyer and the Szász-Mirakjan-Durrmeyer operators in the L_p-spaces on the half-line.

A formula for the error of finite sinc-interpolation with an even number of nodes

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Recently we gave a formula for the error committed when truncating the sinc-series of a function that does not decrease sufficiently rapidly for the discarded tails of the series to be negligible. The main part of the formula is a polynomial in the distance between the nodes whose coefficients contain derivatives of the function at the extremities.

The middle term of a sinc-series usually corresponds to the origin, so that its symmetric truncation contains an odd number of terms, one for every node. In our talk we give the formula for the case of an even number of nodes.

Tree-based Adaptive Learning Algorithms

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We consider an adaptive process of solving a nonlinear approximation problem in which the adaptive decisions are organized in a tree. In each step the decision is based upon the information gained via certain queries about the function f to be approximated. The function f itself is known only through a number of samplings drawn independently according to an unknown probability measure for which f is the regression function. Thus, the queries can be based only on these m samplings and the approximation results should be statistical in nature. Let N be the number of parameters used to define the approximant. It should be clear that a smaller N would result bad approximation, while if N is too large the probability of a failure would be unacceptably high. The goal is to design an algorithm, linear in N, which finds a near-best approximant with high probability.

A prediction scheme for the adaptive approximation of nonlinear functions of wavelet expansions

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A core ingredient of adaptive wavelet methods for nonlinear operator equations is the adaptive evaluation of nonlinear functions. We present an efficient adaptive method for approximately evaluating nonlinear functions of wavelet expansions using semi-orthogonal spline wavelets. Solving the two tasks of predicting an index set for the approximation in terms of wavelets, as well as the approximative computation of the corresponding wavelet coefficients, we are able to achieve the desired accuracy. The computational complexity of the proposed method has the same asymptotic behavior as the best n-term tree approximation.

On the construction of optimal cubature formulas for r times differentiable functions on a ball

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We consider the problem of construction of the optimal formula of approximate integration along the ball $B[0,R]$ in $\mathbf{R}^d$ which uses as information integrals of the function along n spheres centered at the origin and contained inside this ball. The integrands belong to the class of continuous functions on $B[0,R]$ whose generalized radial derivative of order $r \geq 1$ is in the unit ball of the weighted L_p-space on $B[0,R]$, $p \geq 1$, with the weight $|x|^{1-d}$.

This problem is reduced to the problem of the best approximation of the power function $y(t) = t^{r+d-1}$ by linear combinations of truncated powers of order $r-1$ in the space $L_{p'}[0,R]$.

Using known results on spline approximation we study the asymptotic behavior as n gets large of the error of the optimal cubature formula on the class of functions described above.

Exact solution for the case $r = 1$ and $p = 1$ has been obtained earlier by the authors.

Asymptotic results for the weighted best-packing problem on rectifiable sets

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We consider the limiting case of the problem of minimization of the weighted Riesz energy on compact sets $A \subset \mathbf{R}^m$ which can be obtained as an image of a bounded set from $\mathbf{R}^d$ with respect to a Lipschitz mapping ($1 \leq d < m$). The asymptotic behavior of the quantity

$$\delta_N^w(A) := \max_{x_1,\ldots,x_N \in A} \min_{i \neq j} w(x_i, x_j)\, |x_i - x_j|$$

as N gets large is obtained for weights $w(x,y) \geq 0$ which are continuous on the diagonal $D(A)$ of $A \times A$ at almost every point with respect to the d-dimensional Hausdorff measure, bounded in some neighborhood of $D(A)$ and separated from zero on any closed subset of $A \times A$ disjoint with $D(A)$.

We also find the weak* limit distribution of asymptotically optimal sequences of N-point configurations as N gets large.

These results are also obtained for the case when w has a finite number of poles of order less than one on $D(A)$.

The support of the limit distribution of optimal Riesz energy points on sets of revolution in $\mathbb{R}^3$

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Let A be a compact set in the right-half plane and $\Gamma(A)$ the set in $\mathbb{R}^3$ obtained by rotating A about the vertical axis. We investigate the support of the limit distribution of minimum energy point charges on $\Gamma(A)$ that interact according to the Riesz potential $1/r^s$, $0 < s < 1$, where r is the Euclidean distance between points. Potential theory yields that this limit distribution coincides with the equilibrium measure on $\Gamma(A)$ which is supported on the outer boundary of $\Gamma(A)$. We show that there are sets of revolution $\Gamma(A)$ such that the support of the equilibrium measure on $\Gamma(A)$ is not the complete outer boundary, in contrast to the Coulomb case $s = 1$. However, the support of the limit distribution on the set of revolution $\Gamma(R + A)$ as R goes to infinity, is the full outer boundary for certain sets A.

A Normalization for the Riesz N-energy

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We consider a compact set A of positive and finite d-dimensional Hausdorff measure embedded in R^n. Potential theory has addressed the problem of finding a unique probability measure which minimizes the Riesz s-energy when s is less than d. In the case when s is equal to or greater than d, established techniques break down because the Riesz kernel is no longer integrable. For the case when d equals n, we provide a modified energy which is finite and has a unique minimizing probability measure. Further, we show that the minimizing probability measures for s less than n converge in the weak-star topology to our minimizer as s approaches n from below.

Approximation processes for resolvent operators

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In collaboration with Albanese and Mangino (2006, 2007), we have considered the possibility of representing the resolvent operator of a C_0-semigroup by means of iterates of suitable approximation processes. More recently in collaboration with Tacelli (2007) we have introduced suitable approximation processes for the resolvent operator and we have studied their convergence providing also some quantitative estimates. The representation formulas obtained in this manner are particularly useful in studying qualitative properties of the semigroups, as we shall indicate with some applications.

Some consequences concerning with cosine functions is also considered.

Properties of regularization operators in learning theory

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We discuss the properties of a large class of learning algorithms defined in terms of classical regularization operators for ill-posed problems. Popular algorithms from this class, such as Regularized Least-Squares (RLS) and Landweber Algorithm were studied in [1] and [2]. In particular, in [1], a minimax analysis was performed, and it was showed that RLS attains optimal rates of convergence over a suitable family of priors. We describe similar optimality results for general regularization operators in the supervised setting of learning theory. We show that suitable data-dependent criteria for the choice of the regularization parameter enforce adaptation over the considered family of priors.

[1] A. Caponnetto and E. De Vito. Optimal rates for regularized least-squares algorithm. Foundations of Computational Mathematics, 2006.

[2] Y. Yao, L. Rosasco and A. Caponnetto. On early stopping in gradient descent learning. to appear in Constructive Approximation.

n-Dimensional Spaces with Maximal Projection Constant

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The case $n = 1$ is the Hahn-Banach theorem. We will discuss n larger than 1 and show in particular that the maximal projection constant for $n = 2$ is $4/3$ in case the field is real.

Tight Wavelet Frames from Subdivision

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We show how to construct multivariate tight wavelet frame decompositions for scalar and vector subdivision schemes with nonnegative masks. The constructed frame generators have one vanishing moment and are obtained by factorizing certain positive semi-definite matrices. The construction is local and allows us to obtain framelets even in the vicinity of irregular vertices. Constructing tight frames, instead of wavelet bases, we avoid extra computations of the dual masks. In addition, the frame decomposition algorithm is stable as the discrete frame transform is an isometry on ℓ_2, if the data are properly normalized.

The Discovery of the Goldbach Conjecture Code and the Proof of Goldbach Conjecture

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It has been discovered in my research that there is indeed a hidden and unknown real number (called Goldbach conjecture code) in the matching situation of Goldbach conjecture. The discovery of Goldbach conjecture code makes the irregular order in the conjecture be able to be transformed into very regular, so that it can be revealed the law of Goldbach conjecture, or the regulation of distributions of Goldbach conjecture probabilities, exists. It is based on the law, a exactly quantitative equation, that the proof of Goldbach conjecture could be really and completely proved by the probability method and basic mathematical principles and exact quantitative analytical methods. The proved exact mathematical expression of Goldbach conjecture theorem is:

$$M_1 = \frac{A_1 A_2}{N/2 - d} \geq \frac{A_1 A_2}{N/2} \geq 1,$$

or

$$M_1 \geq \frac{1}{\ln(N/2)} \left(\frac{N}{\ln N} - \frac{N/2}{\ln(N/2)} \right) \geq 1, \quad N \geq 30,$$

in which, M_1 is the number of ways to write any even number greater than 2 as a sum of two primes, N is any even number greater than 2 (or $N \geq 30$), A_1 is the number of primes which are less than or equal to $N/2$, A_2 is the number of primes which are greater than or equal to $N/2$ but less than or equal to $N-1$, and d is Goldbach conjecture code $(-N/2 < d < N/2)$.

The proof of Goldbach conjecture reveals that one of the features of prime distribution regulation is the partial symmetric distribution which is related to even number and infinite with even numbers infinite big.

Lagrange Polynomial Interpolation

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Lagrange polynomial interpolation at some well-known nodes, such as zeros of the Chebyshev polynomials of first and second kinds, was extensively investigated. In this paper, we provide a generalization of such investigations and study the Lagrange polynomial interpolation at a special class of nodes. Each number in a closed interval of the real line corresponds to a set of nodes in this class. Many well-known nodes fall into this class. In particular, zeros of the Chebyshev polynomials of first and second kinds, and Chebyshev extrema are the particular cases in this class. The equidistant nodes can be viewed as a limiting case in this class.

From Interpolating Subdivisions to Representation of Scattered Data

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The mathematical contents of this presentation represent selected samples of recent results obtained in an on-going joint research project with Qingtang Jiang on surface subdivisions with matrix-valued templates. In this lecture, we will introduce this topic by extending the cubic B-spline refinement equation to the bivariate setting, thereby arriving at some re-formulation of Loop's scheme with certain degree of freedom, which allows us to extend Loop's scheme to interpolating surface subdivisions, without increasing the cubic polynomial degree and decreasing the C^2 smoothness property. Subdivision templates for extraordinary vertices of arbitrary valences are derived accordingly. An example from our non-spline results to achieve one-ring templates will also be discussed. In a joint work with Wenjie He, the subdivision templates for this example are adopted to construct minimum-supported basis functions for bivariate scattered data interpolation.

Advances on Dislocation properties of Energy Minimizing Configurations, Manifold Image Learning and Applications

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Consider the problem of minimizing the energy of a finite number of fixed particles on a compact set in Euclidean space which interact pairwise via a symmetric, positive definite kernel.

(1) We will begin by discussing recent results of the author and his collaborators on how this problem relates to Thompson's best packing problem and the distribution of charges on Wigner crystals. (2) Next we consider the related problem of learning meaningful descriptions of a large number of data points which arise in ultrasound images of the nerve plexus. We establish the equivalence of energy and discrepancy in this case and prove corresponding bounds for fill distance (mesh norm) and diffusion metrics for the related class of reduced learning kernels. (3) We conclude with a brief discussion on some other interesting connections of this circle of ideas to imaging of neutrons in biochemical assays and seperation estimates for Riesz configurations.

Applications of Curvelets in Computer Vision

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Curvelets [1] are a recent construction of a transform with good time-frequency-direction localization. As a tight frame, it provides a stable representation of L_2 functions. There are also the closely related constructions of Contourlets [7] and Shearlets [3]. In [2], the authors show similarities between Curvelets and new models of the human vision system developed by researchers working in Natural Scene Statistics. So, the obvious question is: can Curvelets be actually used as the low-level vision building blocks for high-level computer vision algorithms? Indeed, usually the first part of a paper describing a computer vision algorithm(e.g. [4], [5], [6]) deals with low-level vision aspects: Which local filtering process to use? How to incorporate scale invariance? How to quantify saliency? How to capture directionality? We note that in many cases the signal cannot be reconstructed from the responses of the low-level vision system being used. In the talk we explain how to use Curvelets as computational framework for low-level vision and present some initial findings of work in progress.

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3. K. Guo, G. Kutyniok, and D. Labate, Sparse Multidimensional Representations using Anisotropic Dilation and Shear Operators, Wavelets and Splines (Athens, GA, 2005), Nashboro Press, Nashville, TN (2006), 189-201.

4. T. Kadir and M. Brady, Saliency, Scale and Image Description, Comp. Vision 45 (2001), 83-105.

5. D. G. Lowe, Object Recognition from Local Scale-Invariant Features, Proc. IEEE Comp. Vision 2 (1999), 1150-1157.

6. K. Mikolajczyk, A. Zisserman and C. Schmid, Shape recognition with edge-based features, Proceedings of the British Machine Vision Conference (2003).

7. D. Po and M. Do, Directional multiscale modeling of images using the contourlet transform, IEEE Transactions on Image Processing 15 (2006), 1610-1620.

The method of cyclic projections: the rate of convergence

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The convex feasibility problem is to find a point in the (nonempty) intersection of a finite number of closed convex sets in a Hilbert space. This formulation includes many interesting and important problems in a variety of areas such as solving linear equations and inequalities, linear prediction theory, image restoration, and computed tomography. The method of cyclic projections is an iterative technique for finding such a point by cycling through projections onto the individual sets which comprise the intersection. In this talk, we will describe what is known about the rate of convergence of the method.

A Taste of Compressed Sensing

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Discrete Compressed Sensing samples a discrete signal $x \in \mathbb{R}^N$ by n linear measurements each an inner product of x with a vector from $\mathbb{R}^N$. If n is the number of measurements allocated to the sensor then the whole process can be represented by an $n \times N$ matrix Φ. The vector $y = \Phi(x)$ represents the n samples we have about x. A decoder Δ is a mapping from $\mathbb{R}^n \rightarrow \mathbb{R}^N$. The vector $\Delta(\Phi(x))$ is the approximation we have to x from the information y. We will discuss how well such an encoding-decoding scheme can perform given n, N. In particular, we ask whether there is a value k such that $\|x - \Delta(\Phi(x))\|_{\ell_p} \leq C_0 \sigma_k(x)_{\ell_p}$, where σ_k is the error in k-term approximation. If so we are interested in the largest possible value of k given the vector length N and the information budget n. We shall see that the answer to this question changes whether we want such performance with certainty or just with high probability.

Some examples of orthogonal matrix polynomials satisfying odd order differential equations

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It is well known that if a family orthogonal scalar polynomials with respect to a positive measure (supported on the real line) forms a set of eigenfunctions of a finite order differential operator, then its order has to be even. This property no longer holds in the case of matrix orthogonal polynomials. The subject of this communication is to present examples of weight matrices having orthogonal polynomials which are eigenfunctions of certain differential operators of odd order. The weight matrices are of the form

$$W(t) = t^{\alpha} e^{-t} e^{At} t^{B} t^{B^*} e^{A^* t},$$

where A and B are certain (nilpotent and diagonal, respectively) $N \times N$ matrices. These weights are the first examples illustrating this new phenomenon and are not reducible to scalar weights.

Also, the behavior of the algebra of differential operators having these families of orthogonal matrix polynomials as eigenfunctions is analyzed for the 2×2 case. In the scalar case, the algebra reduces to the associated second order differential operator and any polynomial in that operator. However, in the matrix setting we have discovered families of orthogonal polynomials which are common eigenfunctions for several linearly independent second order differential operators.

On an energy problem with Riesz external field

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In this talk we consider the minimal s-energy problem on the unit sphere in $\mathbf{R}^{d+1}$ in the presence of an external field, induced by a point charge. The model interaction is that of Riesz kernels $1/r^s$ with $d-2 < s < d$. The support and the density of the corresponding equilibrium measure is given in terms of the distance R from the point charge to the origin and its charge m. The solution utilizes a continuous version of the iterated balayage algorithm.

Obreshkov interpolation polynomial and Hermite corner cutting

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Obreshkov interpolation polynomial has been used for better integration formulae and for efficient numerical solutions of differential equation. In any case, it provides more precise approximations and can be useful in geometric modeling. For every positive integer d, we consider two specific Hermite subdivision schemes of degree d. The first one is interpolatory by solving a Hermite problem involving the values of a function and of its derivatives of order $\leq d$ at two points. The second one is noninterpolatory and is a kind of corner cutting which is a generalization of the well known Chaikin's algorithm. For each of these schemes, for $d = 1, 2, 3$, we compute a subdivision matrix and we check that its spectral radius is < 1.

Thin plate splines on the group $SO(3)$

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In the d-dimensional Euclidean space the thin plate splines introduced by Jean Duchon constitute a powerful tool for the interpolation of scattered data points and for the approximation of functions.

In the talk we will extend the concept of thin plate splines to the rotation group $SO(3)$. We will construct splines which interpolate scattered data points on the manifold $SO(3)$ and which simultaneously minimize the seminorm of Sobolev spaces. We will show that the thin plate splines on $SO(3)$ similarly to the classical case can be regarded as fundamental solution of a differential equation. Finally some of these splines will be presented explicitly.

Generalized Native Spaces

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Given a positive definite kernel function, we will generalize the usual radial basis function Hilbert type of native space construction in order to create L_p based Banach space types of native spaces. We will obtain generalized generic power function error estimates without using Hilbert space projection methods.

Bivariate Splines for Functional Regression Models Over a Unit Square

B. Ettinger*, S. Guillas, and M.-J. Lai

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We explain how to use bivariate splines for the regression analysis of functional data located on a 2D domain. We illustrate the approach on the functional linear model and autoregressive model. We apply the methodology to the prediction of functions over the unit square.

Bivariate orthogonal polynomials in the Lyskova class

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Classical orthogonal polynomials in two variables can be characterized as the polynomial solutions of a matrix second orther partial differential equation involving matrix polynomial coefficients. In this work, we study classical orthogonal polynomials in two variables whose partial derivatives satisfy again a second order partial differential equation of the same type.

Approximation on homogeneous spaces by de la Vallée Poussin type operators

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Let S^d be the unit sphere embedded in R^{d+1} with surface measure μ_d. We would like to approximate functions $f \in L^p(S^q, \omega_q)$ by convolution type operators $A_n f = K_n * f$, where K_n is a suitable kernel. For this it is necessary to have operators which are uniformly bounded with respect to n, and for which we have concrete estimates for the operator norm. Moreover, we would like to have operators which reproduce spherical polynomials up to certain degree. One important application is for example the construction of positive quadrature formulas on the sphere (see H.N. Mhaskar, F.J. Narcowich, J.D. Ward, Spherical Marcinkiewic-Zygmund Inequalities and positive Quadratur, Math. Comp., 70, 2000) We show how to construct such operators in a quite general setting not restricted to the sphere. Finally we show how this can be applied for the construction of quadrature rules.

A smoothed distance function based on mean value interpolation

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Mean value interpolation has recently emerged as an attractive way to interpolate a function defined on the boundary of a planar region enclosed by one or more boundary curves, either smooth or polygonal. In this paper, we propose a 'weight' function as the reciprocal of the denominator in the rational expression for the interpolant, and show that

it behaves like a smoothed version of the distance function. We derive several properties of this weight function, including upper and lower bounds, its normal derivative at the boundary, and a minimum principle. We also give an algorithm for computing partial derivatives of any order. We apply the weight function to the Finite Element solution of elliptic PDE's using the web-spline method of Hollig, Reif, and Wipper.

Best conditioned bases in connection with minimal projections

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Norms of projections and interpolating projections can be estimated in terms of bases condition numbers. These considerations turn into inequalities involving projection constant, Banach–Mazur distance, and interpolating projection constant. Some situations where the inequalities can or cannot become equalities are investigated. Examples in small dimension are presented along the way.

Sobolev-type Approximation Rates for Divergence-free and Curl-free RBF Interpolants

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Recently, error estimates have been made available for divergence-free radial basis function (RBF) interpolants. However, these results are only valid for functions within the associated reproducing kernel Hilbert space (RKHS) of the matrix-valued RBF. Functions within the associated RKHS, also known as the "native space" of the RBF, can be characterized as vector fields having a specific smoothness, making the native space quite small. In this talk we present Sobolev-type error estimates when the target function is less smooth than functions in the native space.

Invariance Theorems of Approximation for Locally Compact Groups

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Let F be a closed linear subspace of a Banach space F and let $\{T_s\}_{s\in G}$ be a group of continuous linear operators T_s mapping F into F, where G is a locally compact topological

group. We prove that if $f \in F$ is invariant under $\{T_s\}_{s \in G}$, then under some conditions on f, F, B, and G, there exists an element $g^* \in B$ of best approximation to f that has the same property. In addition, we establish a version of this result when F is a rearrangement-invariant space and B a subspace of entire functions of exponential type from F. Two examples of application of invariance theorems to harmonic approximation are discussed as well.

Smoothness analysis of nonlinear multivariate Subdivision Schemes

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We present a method to analyze the smoothness of certain nonlinear multivariate Subdivision Schemes defined on a manifold or Lie group. We also study some relations to multiscale data processing.

WEB-Spline Finite Element Method

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This research is an investigation the use to weighted extended B-splines (WEB-Splines) as a basis for the finite element method. Traditionally the finite element method involves creating a mesh grid to discretize the domain of the problem which adheres closely to the boundary. This is a very time consuming and memory intensive step in the process. WEB-splines alleviate the need to create a mesh grid, and rely upon a uniform grid across the domain. This fact, in addition to the approximation properties of B-splines make the method highly desirable alternative to more tradtional finite elements.

Thin Plate Spline Approximation in the Disc

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The prevailing approach in obtaining error estimates for thin plate spline approximation (a prominent type of RBF approximation) involves expressing interpolants as the solution of a minimization problem (minimizing the seminorm of the “Native Space”, which

for thin plate spline approximation is the homogeneous Sobolev space $\dot{H}^2(\mathbb{R}^2)$). The current state of the art for thin plate spline approximation delivers optimal error bounds when the error is measured in L_2, but when the error is measured in L_p for $p \neq 2$ we are confronted with one of the following two challenges. Either:

1. to determine the saturation rate, because the known rate of approximation is lower than a theoretical upper bound, or
2. to increase the space on which the saturation rate is attained, because it holds for a too-small class of functions.

A further challenge stems from the well-known fact that the error is non-uniform over the underlying domain. It is much larger in a small neighborhood of the domain's boundary, and these "boundary effects" are responsible for a dramatic loss of approximation power when compared to the shift invariant setting.

In this talk, we discuss a new thin plate spline approximation scheme for functions defined on the unit disc in $\mathbb{R}^2$. Based on an integral representation involving low order layer potentials, and utilizing direct approximation procedures that avoid native spaces, we have shown that optimal approximation rates can be obtained for functions coming from large smoothness spaces and that negative boundary effects can be overcome at no additional computational cost, resulting in a striking increase in approximation order.

Algebra and combinatorics of box splines

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We explore connections between enumerative combinatorics, commutative algebra, and box spline theory.

A Subdivision Scheme for Curves

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In this work we extend the corner cutting Chaikin subdivision scheme for control points to a subdivision scheme for non-intersecting, uniformly oriented, 3-D control curves. Our scheme replaces averages between two control points by similar weighted averages between two neighboring curves, based on a geometric correspondence between points on the two curves. The resulting scheme converges and generates C^1 surfaces. Examples demonstrating surfaces generated by our scheme will be provided.

Three-pencil lattices on triangulations

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In contrast to the univariate case, uniqueness of the solution of a multivariate Lagrange polynomial interpolation problem depends not only on the fact that interpolation points should be distinct but also on their geometry. Lattices are perhaps the most often used configurations of prescribed interpolation points.

In this talk, three-pencil lattices on triangulations will be considered. The explicit representation of a lattice, based upon barycentric coordinates, will be presented. This enables us to construct lattice points in a simple and numerically stable way and carries over to triangulations in a natural way. The construction is based upon group action of S_3 on triangle vertices, and the number of degrees of freedom is equal to the number of vertices of the triangulation.

Weighted Spline Wavelets

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We introduce the concept of weighted wavelets which are wavelets constructed by using a lifting scheme and a weighted inner product. Furthermore we present a method to get "lazy" wavelets which are used to construct weighted wavelets for periodic B-splines. Finally these weighted wavelets are applied to the compression of implicitly defined curves.

Local variational spline and subdivision curves and surfaces

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We will discuss two problems: local variational spline interpolation and local variational subdivision, for both curves and surfaces. We show that the error between local and global variational spline and subdivision interpolants decay exponentially over a fixed interval as the support of local interpolants increase. By piecing together these locally defined splines, one can obtain a very good approximation of the global variational splines and subdivision schemes. Moreover, we observer that n-point schemes are the result of the local variational subdivision schemes.

Multiwavelet Decompositions for Denoising and Pattern Matching in an Element-Recognition Application

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This talk will show how the author has used multiwavelets to enhance the ability of Western Kentucky Universitys Applied Physics Institute to detect compounds using their method of pulsed fast/thermal neutron analysis. After bombarding an unknown substance with neutrons, their machinery detects gamma-ray emissions at different energy levels. Substances are detected by noting the Gaussian bumps in the reading that are indicative of certain elements. This is complicated by a large amount of distortion caused by the neutron bombardment, and by the fact that many elements have multiple bumps that overlap those of other elements. The operator uses some numerical calculations to determine the elemental composition of the compound, but those calculations are based on values set by the experienced human operator, leaving the results open to interpretation.

The author has used an orthogonal scaling vector of four compactly-supported functions, developed by Donovan, Geronimo, and Hardin in 1999, that generates a space containing square-integrable cubic splines on integer knots. Orthogonality is maintained when the scaling functions are restricted to a unit interval with integer endpoints, allowing the construction of boundary scaling functions and wavelets. The talk will illustrate how the wavelet decompositions of the signal and ideal signals for each element are used to find a completely objective best-fit solution from a library of known compounds.

Time frequency representations of almost-periodic function

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In this paper, we characterize the space of almost periodic (AP) functions in one variable using either a *Weyl-Heisenberg* (*WH*) system or an *affine* system. Our observation is that the sought-for characterization of the AP space is valid if and only if the given *WH* (respectively, *affine*) system is an $L_2(\mathbb{R})$-frame. Moreover, the frame bounds of the system are also the sharpest bounds in our characterization. This draws an intriguing and quite unexpected connection between $L_2(\mathbb{R})$ representations and AP representations.

Markov-type inequalities for multivariate polynomials in L^p norm

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Markov-type inequalities provide estimates for magnitude of derivatives of polynomials on the given set provided that the size of the polynomials on this set is known. The classical univariate results proved for C and L_p-norms provide the order of magnitude $O(n^2)$, where n is the degree of polynomials. Similar results have been obtained in the sup norm for multivariate polynomials on convex bodies (and more generally sets without cusps), but the methods used to handle the multivariate polynomials in C-norm were not suitable for the L_p-case. In this talk we shall discuss Markov-type inequalities for multivariate polynomials in L_p-norm. The order of magnitude $O(n^2)$ will be shown to hold for two categories of sets in d-dimensional space: A) convex bodies, and B) star-like domains with smooth boundary.

Shearlets: A Wavelet-Based Framework for Geometric Multiscale Analysis

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Nowadays we are confronted with a deluge of multidimensional data, which calls for the development of highly efficient tools for analyzing special geometrical features preferably in a multiscale fashion. One main focus of current research is on the development of directional representations which precisely detect orientations of singularities like edges in a 2-D image while providing optimally sparse representations. Many approaches have been untertaken during the last years, among which were the ridgelets, the curvelets and, recently, the shearlets.

In the first part of this talk we will give an introduction to the theory of shearlets. The shearlet systems are the first directional representation systems, which not only precisely detect directions in the sense of resolving the wavefront set and providing optimally sparse representations, but moreover possess a rich mathematical structure similar to wavelets. In particular, shearlets are affine systems, i.e., they are generated by dilating and translating one single generating function, where the dilation matrix is the product of a parabolic scaling matrix and a shear matrix. We will also see that shearlet systems can be regarded as generated by a unitary representation of the so-called shearlet group, which, for instance, provides the possibility to employ the theory of uncertainty principles to study accuracy of the shearlet parameters.

In the last part of the talk the association of shearlet systems with a multiresolution analysis will be discussed. We will present some very recent results on shearlet subdivision schemes leading to a fast algorithm for shearlet decomposition using FIR-filters.

A geometric approach to irregular wavelet frames

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In wavelet analysis, irregular wavelet frames have recently come to the forefront of current research due to questions concerning the robustness and stability of wavelet algorithms. A major difficulty in the study of these systems is the highly sensitive interplay between geometric properties of the sequence of time-scale indices and frame properties of the associated wavelet system.

In this talk we will introduce the notion of affine density and show that it is a highly effective tool for examining the geometry of sequences of time-scale indices. In particular, we will derive a necessary condition on the relationship between the affine density, the frame bounds, and the admissibility condition for an irregular wavelet frame. Several implications of this relationship will be studied. For instance, this result reveals one reason why wavelet systems do not display a Nyquist phenomenon analogous to Gabor systems, a question asked in Daubechies' Ten Lectures book. It also implies that the affine density of the set of time-scale indices associated with a tight wavelet frame has to be uniform. Finally, we show that affine density conditions can even be used to characterize existence of wavelet frames, thus serving in particular as sufficient conditions.

Multivariate Splines for Data Fitting and Approximation

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I will first survey the recent results on multivariate splines for scattered data fitting and approximation, i.e., the results how to compute such data fitting splines and how well these splines approximate the given data. Then I will explain some new result on data fitting and approximation when data have some special properties and structures.

Quadrature formulas and localized linear polynomial operators on the sphere

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We describe and compare numerical algorithms for construction of quadrature formulas on Euclidean spheres, exact for spherical harmonics of a high degree. Our formulas are based on scattered sites; i.e., in constrast to such well known formulas such as Driscoll-Healy's, we need not choose the location of the sites in any manner. We are able to construct formulas exact for spherical harmonics of degree 178. We also demonstrate the use of these formulas in constructing localized, linear, quasi-interpolatory polynomial operators based on scattered sites. The approximation and localization properties of our operators are studied theoretically in deterministic as well as probabilistic settings. Numerical experiments are presented to demonstrate their superiority over traditional least squares and discrete Fourier projection polynomial appproximations.

On Multidimensional Refinable Functions

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Multidimensional refinable functions have many applications in diversified disciplines such as multidimensional multirate perfect reconstruction filter bank design and computer-aided geometric design. We are interested in a special family of functions in R^s that are refinable with respect to any expansive sampling matrix A satisfying $A^s = \pm a I_s$, where $a = \det A$ and I_s is the identity matrix of order s. We will point out certain families of refinable box-splines with finite masks. We will also demonstrate, in particular, some bivariate and trivariate such refinable functions and highlight some of their interesting applications.

Characterization of the Total Variation of Piecewise Smooth Functions by Shearlets and Related Variational Methods

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Variational methods have been extremely successful in a wide variety of image restoration problems and exhibit the solution of these problems as minimizers of appropriately chosen functionals which often involve the total variation. We show that 'shearlets' completely identify the discontinuity curves of a piecewise smooth function in the 2 dimensional case and this provides a shearlet characterization of the total variation of piecewise smooth functions. Afterwards, we prove that variational models involving the total variation such as the total variation denoising and the Mumford-Shah free-boundary segmentation model can be reformulated in terms of shearlet decomposition. Finally, we discuss implementation issues of our approach.

Interpolation by entire functions

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The topic of this talk is interpolation by entire functions and best approximation by functions of finite type on the real line (one-sided and ordinary). Such a best approximation can often be described by the sign changes of the error function.

This suggests the following problem: Given a real function $f(x)$ and a set of interpolation nodes S on the real line satisfying $\sum_{a\in S} a^{-2}$, form the product

$$F(x) = \prod_{a\in S} (1 - z/a) \exp(z/a)$$

and define an entire interpolation $A(x)$ to $f(x)$ that satisfies $f(x) - A(x) = F(x)H(x)$ with a function $H(x)$ which we would like to be non-negative.

The starting point is a representation $1 = F(x)L[g](x)$ in a vertical strip (L denotes the two-sided Laplace transform, hence F determines g). In certain cases this leads to a representation

$$A(x) - f(x) = F(x)L_0[g_1 - g_2](x)$$

with explicitly computable g_1 and g_2. I will mainly limit myself to special functions $f(x)$ like $x_+^n, \exp(-x^2), \cosh(x)$, and describe how far the procedure outlined above goes through, and where the obstacles are to generalize this to function classes.

Prewavelet Solution to Poisson Equations

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Finite element method is one of powerful numerical methods to solve PDE. Usually, if a finite element solution to a Poisson equation based on one level triangulation of the underlying domain is not accurate enough, one will discard the solution and then refine the triangulation and compute a new finite element solution at the refined level. We propose a prewavelet method by keeping the original finite element solution and then adding a prewavelet solution to obtain an approximation of the refined level finite element solution. To increase the accuracy of numerical solution to Poisson equations, we can keep adding prewavelet solutions.

Our prewavelets are orthogonal in the H^1 norm and they are compactly supported in a triangular domain and they are compactly supported except for one globally supported basis function in a rectangular domain. We have implemented these prewavelet basis functions and used them for numerical solution of Poisson equation with Dirichlet boundary conditions. Numerical simulation demonstrates that our prewavelet solution is much efficient than the standard finite element method.

Generalized Bivariate Quadratic B-splines: An Empirical Study

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Generalizing classic univariate B-splines into the multivariate setting is a long-studied problem in spline theory. Previously, Neamtu provides an elegant solution to this problem using *Delaunay configurations*. This solution, although elegant, has short-comings: The use of Delaunay configurations not only involves fairly complicated geometric computations but also places what are perhaps excessive restrictions on the spline spaces that can be constructed. Therefore, to improve, in a previous work, we generalize Neamtu's construction for the bivariate quadratic and cubic cases, so that the Delaunay criteria are no longer needed. Here, as a follow-up to that work, we demonstrate that our generalization is useful in practice. In GIS applications, input data often contain both scattered points and line segments. The latter represent sharp features on the terrain. While most smooth fitting methods do not use the line segments, we show how to do so with our quadratic B-splines. In CAD applications, a modeler often creates a single surface by sewing together a number of spline patches. We show how to join patches of quadratic box spline surfaces smoothly by blending them around the patch boundaries.

A New Approach to Universality Limits

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Universality limits involving orthogonal polynomials are important in mathematical physics and random matrices. We discuss a new approach to proving these, involving a localization technique and a smoothing one. These work both in the bulk of the spectrum, and at the edge, and can handle measures that are absolutely continuous only in a small neighborhood of the region where universality is desired.

A Bernstein-Bezier basis for Hermite subdivision with shape constraints on a rectangular mesh

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We study a two parameter version of the Hermite subdivision scheme introduced

by Merrien and Dubuc in 1999 which gives C^1 interpolants on rectangular meshes. By introducing a Bernstein-Bézier basis and a corresponding control grid we can choose the parameters in the scheme so that the interpolant inherits positivity and/or directional monotonicity from the initial data. Several examples are given showing that a desired shape can be achieved even if we use only very crude estimates for the initial slopes.

A general, multipurpose interpolation procedure: the magic points

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Lagrangian interpolation is a classical way to approximate general functions by finite sums of well chosen, pre-defined, linearly independent basis functions. It is much simpler to implement than determining the best fits with respect to some Banach (or even Hilbert) norms. In addition, only partial knowledge is required (here values on some set of points). The problem of defining the best sample of points is nevertheless rather complex and is in general not solved.

In this paper we propose a way to derive a hierarchical family of "*empirical Lagrangian interpolation*" sets of points. We do not claim that the points resulting from the construction explained here are optimal in any sense. Nevertheless, we can state a theorem that proves that, under certain hypothesis, the process does provide a convergent approximation. In addition our approach is very general and simple to implement,. Compared to situations where the best behavior is known (polynomial approximation in 1, 2 or 3 dimensions), it is competitive.

The interest of our approach is that it can be used for approximation by algebraic polynomials, Fourier series, spherical harmonics, spline, rational functions, but also by linear combination of any kind of basis functions.

Diffusion Multiscale analysis on metric measure spaces

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Motivated by applications to regression and approximation on low-dimensional sets in high-dimensional spaces and on graphs, we present a construction of multiscale tight frames on metric measure spaces. Typically we start from the eigenfunctions of a diffusion semigroup on the space, and construct a multiscale by smoothed spectral projections. We show that this allows a characterization of Besov spaces, and also prove localization estimates for the associated kernels. We present an application to automatic handwritten

digit recognition where we show our method outperforms current standard techniques. This is joint work with H. Mhaskar.

Wavelet Decomposition for Adaptive Irregular Grids

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Wavelet decomposition algorithms of singular numerical signals (i.e. numerical signals with fast oscillations of variation velocity of module for difference of neighbour signal values) are developed. Qualitative processing of mentioned signals is used adaptive irregular grid (the irregular grid which is adjusted to characteristics of incoming signals automatically). Algorithms for construction of the grid are represented, relevant formulas of wavelet decomposition and reconstruction are elaborated, calculation stability and approximation properties are investigated. Obtained formulas are easily parallelized.

Shape Preserving Regions for Hermite Subdivision

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In this talk we analyze connections between constrained Hermite subdivision schemes belonging to the two parameters family introduced in [3] and analyzed by several authors, and classical Hermite shape preserving interpolation methods. By means of this analysis, for any set of monotone or convex data we determine (two-dimensional) subsets of the convergence region of the subdivision scheme ensuring monotonicity or convexity preservation.

In addition, we describe a general setting to construct families of shape preserving Hermite subdivision schemes belonging to the Merrien's class. The construction bases on structural properties of four dimensional spaces used in shape preserving interpolation related to the existence of a Bernstein like basis.

The proposed construction includes, as particular cases, the shape preserving families introduced and analyzed in [4] and in [2]. Moreover, it provides new shape preserving families, considering function spaces very popular in the context of shape preserving interpolation as exponential, rational, variable degree polynomial or parametric cubic functions.

Each family depends on one shape parameter. We provide explicit choices for this parameter, we prove the convergence and analyze the performances of the schemes when the shape parameters are selected adaptively during the subdivision process. Being this selection strategy nonlinear, we are dealing with nonlinear subdivisions. It turns out that this nonlinear approach produces limit functions smoother than those built by the corresponding stationary (shape preserving) schemes. In some salient cases the limit consists in a finite, and a priori bounded, number of cubic (or piecewise quadratic) segments.

Finally, we discuss some possible extension of the analyzed subdivision schemes.

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Bernstein-type bases via blossoming

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Bernstein-type bases play a fundamental role in geometric design. Their existence is equivalent to existence of blossoms, from which they naturally emerge, along with the fact they are the optimal normalised totally positive bases. We also show how other classical properties of such bases (such as recurrence relations - differentiation - dimension elevation) are related to blossoms.

From Hermite Subdivision Schemes to Vector Subdivision Schemes via Taylor Polynomials

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A Hermite subdivision scheme $\mathcal{H}$ of degree d is a recursive scheme for computing a function ϕ and its d derivatives $\phi', \ldots, \phi^{(d)}$. The initial state of the scheme is a vector function f_0 from $\mathbf{Z}$ to $\mathbf{R}^{d+1}$. The first component of f_0 is a control value for ϕ, the second component, for ϕ' and so on. The sequence of refinements $f_n : \mathbf{Z} \to \mathbf{R}^{d+1}$, $n > 0$, is recursively defined through a family of $(d+1) \times (d+1)$ matrices $\{A(\alpha) = (a_{ij}(\alpha))_{i,j=0,\ldots,d}\}_{\alpha \in \mathbf{Z}}$, a finite number of them being non-zero, by

$$D^{n+1} f_{n+1}(\alpha) = \sum_{\beta \in \mathbf{Z}} A(\alpha - 2\beta) D^n f_n(\beta), \alpha \in \mathbf{Z}, n \geq 0,$$

where D is the diagonal matrix whose diagonal elements are $1, 1/2, \ldots, 1/2^d$.

If $\mathbf{H}$ is non degenerate and if for every sequence $f_n : \mathbf{Z} \to \mathbf{R}^{d+1}$,

$$f_n^{(i)}(\alpha+1)/2^{in} - \sum_{j=i}^{d} \frac{1}{(j-i)!} f_n^{(j)}(\alpha)/2^{jn} = o(1/2^{dn}) \quad \alpha \in \mathbf{Z}, i = 0, 1, ..., d.$$

then for every $k = 0, \ldots, d$, there exists a unique polynomial $p_k(x)$ of degree k with its coefficient of x^k equal to $1/k!$ such that the vector function $v_k : \mathbf{Z} \to \mathbf{R}^{d+1}$ where $v_k(\alpha) = (p_k(\alpha), \ldots, p_k^{(d)}(\alpha))^T$ is an eigenvector of $(A(\alpha - 2\beta))_{\alpha,\beta \in \mathbf{Z}}$ with the eigenvalue $1/2^k$ (Spectral Condition)

Conversely, if $\mathbf{H}$ satisfies the spectral condition, we can associate a vector subdivision scheme $\mathbf{S}$. If moreover $\mathbf{S}$ is C^0 with equal components then the Hermite subdivision scheme is C^d.

Polynomials Orthogonal over a Level Set of a Hypocycloid

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A hypocycloid H_m of m-cusps ($m \geq 3$) is the image of the unit circle by the mapping

$$\psi(w) = w + \frac{1}{(m-1)w^{m-1}}.$$

Thus, H_m is a piecewise analytic Jordan curve having outer cusps at each of its m corners $me^{2\pi ki/m}/(m-1)$, $k = 0, .., m-1$, and ψ maps the exterior of the unit circle conformally onto the exterior of H_m. For $R > 1$, let L_R be the level curve $L_R := \{\psi(w) : |w| = R\}$, and let G_R be the domain interior to L_R.

We consider the sequence $\{P_{n,R}(z)\}_{n=0}^{\infty}$ of polynomials that are orthonormal with respect to area measure over G_R. In this talk we will describe the asymptotic behavior of these polynomials and that of their zeros, and we will highlight how such a behavior depends on the magnitude of the level R of the orthogonality domain G_R.

L-CAMP and other types of pyramidal representations based on sampling

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Wavelet pyramids are obtained by extracting detail coefficients from an MRA (Gaussian) pyramid. Typically, the MRA resolution layers are generated by successive application of the operation $y \mapsto (c * y)_\downarrow$ where c is a low-pass filter, and $\downarrow$ denotes the down-sampling operation. We observe two possible shortcomings:

- We always have to compute coarse coefficients. This becomes costly when the size of data is large. Consequently, there is high overhead when one is only interested in detail coefficients at coarse resolutions.

- There is excessive blurring of data due to the repeated application of low-pass filtering.

In this talk, we introduce wavelet-like pyramidal representations that are based on blurring-free MRA, i.e., on the successive process $y \mapsto y_{\downarrow}$. This avoidance of the convolution step is especially important when the size of the data set is large, e.g., in high dimensions. Using L-CAMP methodology, we construct very local high-performance systems in surprisingly simple manner. Some of our constructions are the most local among the systems that we know from the literature. Our main results concern the theoretical performance of this approach, i.e., identifying the smoothness classes that are encoded correctly by these representations. We show that the space L_2 that plays the role of the "base space" in the known theory for wavelet/framelet systems can be successfully replaced in our setting by the space C_0 of continuous functions that decay at ∞.

A Bernstein theorem for spherical basis functions

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The fitting of scattered data collected at remote ground stations or by satellites is important in geophysical and atmospheric research. Approximation and interpolation using spherical basis functions (SBFs) are increasingly important methods for dealing with such data. In this talk, we will concentrate on inverse (Bernstein-type) Sobolev error estimates for SBF methods. These estimates are established with the help of well-localized, spherical polynomial kernels. The talk is based on work joint with H. Mhaskar, P. Petrushev, X. Sun, J. Ward, and H. Wendland.

Local Lagrange Interpolation with Bivariate and Trivariate Splines

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Local Lagrange interpolation methods for splines on arbitrary triangular and tetrahedral partitions are described. The construction of interpolation points is based on new type of priority principles. This means that the partitions are decomposed efficienly into classes of (splitted and non-splitted) triangles and tetrahedra, respectively. The interpolating splines can be computed locally, with linear complexity and yield optimal approximation order. In the bivariate case, we investigate splines of arbitrary degree, and in the trivariate case splines of special degree. Numerical examples with real world data show the efficiency of the spline algorithms. The methods were developed in cooperation with Chui, Hecklin, Rayevskaya, Schumaker and Zeilfelder.

Problems in Subdivision Theory

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This talk is based on my scattered results on semi-regular and nonlinear subdivision methods and their applications to the analysis of multiscale algorithms, and aims at highlighting some of the partially unsolved problems in this research area. For illustration, I will detail a recent result on multiscale preconditioning with nonconforming finite elements, where convergence of cascade algorithms in Sobolev spaces of non-integer smoothness turned out to be crucial for proving asymptotic optimality, and improving the preconditioning performance as well. As to nonlinear subdivision, where the current focus is on convergence and smoothness issues (see the minisymposium at this conference), I will briefly address the stability problem.

On the Best Relative Approximations of Functional Classes by Splines

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Let L_p $(1 \le p \le \infty)$ be the spaces of 2π–periodic functions $f : \mathbf{R} \to \mathbf{R}$ with corresponding norms $\|\cdot\|_p$, M and M' be the certain subsets of L_p.

The value

$$E(M, H \cap M')_p = \sup_{f\in M} \inf_{h\in H\cap M'} \|f - h\|_p$$

is called the best relative approximation of the set M by the set $H \subset L_p$ in the space L_p.

We denote by W_p^r $(r \in \mathbf{N})$ the class of functions $f \in L_p$ such that $f^{(r-1)}$ $(f^{(0)} := f)$ is locally absolutely continuous and $\|f^{(r)}\|_p \le 1$; W_V^r $(r \in N)$ the class of functions $f \in L_p$ such that $f^{(r-1)}$ is locally absolutely continuous and $\bigvee\limits_0^{2\pi}[f^{(r)}] \le 1$.

In addition, let $S_{2n,r}$ $(n, r \in \mathbf{N})$ be the spaces of polynomial splines of order r defect 1 with knots in the points $k\pi/n$ $(k \in \mathbf{Z})$ and $\varphi_{\lambda,r}(\cdot)$ $(\lambda > 0,\, r \in N)$ be the r–th $2\pi/\lambda$–periodic integral of $\varphi_{\lambda,0}(x) = \operatorname{sign}\sin\lambda x$ with zero mean value on the period.

We consider the behaviour of quantities $E(W_1^r, S_{2n,r+k} \cap M_n W_1^{r+k})_1$ ($r = 2, 3, ...,$ $k = 0, 1, 2, ...$) under various values of M_n.

In particular, it was shown that for $M_n \geq rn^k \|\varphi_{1,r}\|_\infty / \|\varphi_{1,r+k}\|_\infty$ the following correlation is valid.

$$E(W_1^r, S_{2n,r+k} \cap M_n W_1^{r+k})_1 = E(W_1^r, S_{2n,r-1})_1 = \|\varphi_{1,r}\|_\infty / n^r \qquad (1)$$

for all $r = 3, 4, ..., k = 0, 1, ...$ and $n = 1, 2, ...$

In addition, the estimates of M_n for which correlation (1) is not true are obtained.

The problems as to exact order and exact asymptotic (as $n \to \infty$) of these quantities are also studied.

Recent advances on optimal properties of normalized B-bases

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Any space possessing shape preserving representations of curves in CAGD possess a unique basis called normalized B-basis, which can be considered as a generalization of the Bernstein basis and which has optimal shape preserving properties. After recalling other known optimal properties of normalized B-bases, we present new optimal properties of these bases. The use of of normalized B-bases to derive bases for the representation of surfaces with optimal properties is also illustrated. Some open problems are presented.

A Stieltjes function in two variables

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In this work, a Stieltjes function in two variables is studied. This Stieltjes function will be used to characterize classical orthogonal polynomials in two variables.

On dual extremal problems for abstract valued functions

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Special geometric properties of Banach space of functions codomain effect behavior of Banach-space-valued functions significantly. For instance, absence (in general case) of

maximum module principle in its strict form relates to the fact that it holds only for the case when functions codomain possesses specific geometric characteristics.

In the present paper, we study extremal problems for functionals (which were to be described) over spaces H(B) of bounded analytic functions from the open unit circle of a complex plain to a complex Banach space B. In particular, we consider a dual extremal problem for a subspace of H(B) consisting of functions whose nontangential boundary values exist everywhere on the unit circumference and constitute continuous functions there. We investigate the above problem in connection to properties of Banach space codomain of the functions and its ajoint space.

Anisotropic Smoothness Spaces via Level Sets

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It has been understood for sometime that the classical smoothness spaces, such as the Sobolev and Besov classes, are not satisfactory for certain problems in image processing and nonlinear PDEs. Their deficiency lies in their isotropy. The anisotropic generalizations of these spaces also have the deficiency that they are biased in coordinate directions. While they allow different smoothness in certain directions, these directions must be aligned to the coordinate axes. In the application areas mentioned above, it would be desirable to measure smoothness in new ways which would allow one to have more local control over the smoothness directions. We introduce one possible approach to this problem based on defining smoothness via level sets. Our smoothness spaces depend on two smoothness indices (s_1, s_2). The first reflects the smoothness of the level sets of the function, while the second index reflects how smoothly the level sets themselves are changing.

Nonstationary subdivision schemes and totally positive refinable functions

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Total positivity plays a main role in several problems of approximation theory, as well as in CAGD questions [3]. In fact, several approximation operators, based on totally positive functions, display interesting convergence properties and shape preserving behaviour. Examples of totally positive functions are provided by the well known B-splines or, more in general, by a class of parametric refinable functions, which present a great flexibility in several applications [4,5].

The intimate connection between refinability and subdivision schemes is well known and recently suggested the possibility of exploiting nonstationary subdivision schemes for the construction of some totally positive functions [1].

In this paper we present the construction of a class of totally positive functions, obtained by a suitable use of certain nonstationary subdivision schemes. These functions are characterized by having small support and their smoothness can be established a priori.

The same technique can be applied in the bivariate setting. Starting from a class of stationary bivariate subdivision schemes [2], we construct bivariate refinable functions by nonstationary subdivision schemes. As in the univariate case, these functions have small support and prescribed smoothness; moreover, they are bell-shaped.

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Convergence of a nonlinear iterative algorithm for image denoising

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In this paper we present a simple and efficient nonlinear algorithm for recovering of a piecewise constant image from an observed image containing additive noise.

In a first step we apply an adaptive neighborhood filtering scheme. The proposed nonlinear averaging filter is a typical smoothing filter which has no recourse to the input data during the iteration process. In contrast to the usually taken diffusion filters (like Perona-Malik-filter, Charbonnier-filter, regularized TV-filter etc.) the filter in our scheme can be seen as a so-called “robust” filter.

We will be able to show that the iterated application of our proposed neighborhood filter leads to a piecewise constant image, i.e., there exists a partition of the image, such that in each subdomain of the partition the spatial average of the pixel values of all pixels belonging to this subdomain is found. This observation generalizes the known result on convergence of the image to the constant steady-state.

We will show even more, namely that the partition of the image determining the piecewise constant steady-state after an infinite iteration process can already be found after a finite number of iteration steps.

The numerical results impressively show the performance of the algorithm. We compare the proposed algorithm with some other image denoising methods considered in the literature.

L_1-approximation of stationary Hamilton-Jacobi equations

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We describe a nonlinear finite element technique to approximate the solutions of stationary Hamilton-Jacobi equations in one and two space dimensions using continuous finite elements of arbitrary degree. The method consists of minimizing a functional containing the L1-norm of the Hamiltonian plus a discrete entropy. It is shown that the approximate sequence converges to the unique viscosity solution under appropriate hypotheses on the Hamiltonian and the mesh family.

Exponentially localized polynomial frames on the unit interval and the Euclidean sphere

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In this talk we present an exponentially localized polynomial frame on the interval $[-1, 1]$, and on the unit sphere of a Euclidean space. Even though the frame coefficients may be computed using the coefficients of a function in an orthogonal polynomial expansion, the behavior of these coefficients near a point on the interval characterizes the possibility of an analytic continuation of the function in a complex neighborhood of the point in question.

Particularly, we construct a sequence of polynomials which converges uniformly in the order of best approximation to the given function f and geometrically fast at each point where f is analytic.

Our main interest is in the characterization of local smoothness of a function f in terms of the sequence of Fourier coefficients $\{\hat{f}(k)\}$. But from the point of view of computations, we will also describe our results when samples of the functions in question are available, instead of the coefficients.

Our construction allows one to construct exponentially localized kernels based only on some summability estimates. In turn, the localization enables us to obtain a characterization of local Besov spaces on the interval also in the case of some more general systems of orthogonal polynomials.

Results concerning best coapproximation

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The strong uniqueness of best coapproximation of elements in a normed linear space is discussed. The properties of the associated operator are investigated. The strong uniqueness is highlighted by means of relevant examples in specific spaces. Further developments in linear 2-normed spaces are indicated.

A Local Construction for Non-stationary Tight Wavelet Frames

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We present a construction for Tight Wavelet Frames based on per element conditions instead of shift invariance and Fourier analysis, as has been common in many previous constructions. As an example, we apply our method to linear splines over arbitrary refined triangulations to obtain tight framelets with local support and with symmetry properties. We discuss further applications of the construction to splines and spline type subdivision schemes.

An alternative description of oscilatory surfaces

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The movement of an oscilatory surface is described via bilinear De Casteljau quaternionic approximation. That gives us a movie of the oscilating surface starting from some initial data and constraints. Several practical and computational examples will be presented

The complete parameterization of the length eight orthogonal wavelets with no parameter constraints

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Over the years, researchers have tended to use a fairly fixed set of "good" wavelets for their signal processing applications including the Haar wavelet, the Daubechies wavelets

with the maximum number of vanishing moments, or the FBI 9/7 biorthogonal wavelet. To find other lesser publicized wavelets, researchers would be required to perform spectral factorizations or some type of lifting scheme to find the filter coefficients. In this talk, I will give a complete parameterization of the coefficients for the length eight orthogonal filters without any parameter constraints. This is the set of all trigonometric functions which satisfy the necessary condition for orthogonality $|m(\omega)|^2 + |m(\omega + \pi)|^2 = 1$ which includes all of the length eight wavelet filters and tight frames. I will demonstrate how the parameters affect the shape of the scaling functions and the location of zeros for the associated trigonometric polynomial. Finally, I will compare the length eight filters by applying them to an image compression scheme and give examples of the best length eight filter for a given image and fixed compression ratio. As will be illustrated, wavelets with the highest number of vanishing moments are not always the best suggesting the importance of matching the wavelet to the image. With these parameterized formulas, one can test a whole continuum of wavelets in a signal processing application using filters with a variety of smoothness and vanishing moments by simply perturbing the parameter values.

Inverse polynomial images which consists of two Jordan arcs - an algebraic solution

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Let P_n be a polynomial with complex coefficients of degree n and let $P_n^{-1}([-1,1])$ be the inverse polynomial image of $[-1,1]$. Then, in general, $P_n^{-1}([-1,1])$ consists of n Jordan arcs, on which P_n is strictly monotone increasing from -1 to $+1$. If P_n is the classical Chebyshev polynomial $T_n(z) = \cos(n \arccos(z))$ then the inverse image of T_n may be combined to only one Jordan arc, more precisely, $T_n^{-1}([-1,1]) = [-1,1]$. Here we are interested in polynomials for which the inverse image consists of *two* Jordan arcs. This property is equivalent to the existence of a certain quadratic equation (Abel-Pell equation) for the corresponding polynomial. Moreover, the endpoints of the arcs and the polynomial itself may be characterised with the help of Jacobi's elliptic and theta fuctions. Nevertheless, our approach is only algebraic. Starting from a result of Peherstorfer and the author, we derive *one* polynomial equation in terms of the four endpoints of the two arcs. In other words, for every degree n, we can give *explicitly* a homogenous polynomial in four variables, $p(a,b,c,d)$, whose zeros $\{a,b,c,d\}$ are the endpoints of the two Jordan arcs. Moreover, a simple equation for the corresponding extremal points of P_n on its inverse image is derived.

Scattered data approximation on compact groups

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Scattered data approximation is a practical problem that has many applications. Interpolation by translates of positive definite functions turned out to be a good method to handle those problems on different structures. This technique leads to a system of linear equations that has to be solved. We discuss the stability of this system depending on the localization property of the underlying positive definite basis function. Furthermore we propose a method to construct well-localized positive definite functions on semi-simple compact Lie groups. Finally we apply our results to the rotation group $SO(3)$.

Exact Constants in Stechkin-type Inequalities

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The classical Jackson-Stechkin inequality estimates the value of the best uniform approximation of a periodic function f by trigonometric polynomials of degree $\le n-1$ in terms of its r-th modulus of smoothness $\omega_r(f,\delta)$. It reads

$$E_{n-1}(f) \le c_r\,\omega_r\Big(f, \frac{2\pi}{n}\Big),$$

where c_r is *some* constant that depends only on r. It was known that c_r admits the estimate $c_r < r^{ar}$ and, basically, nothing else was known about it.

The main result of this paper is in establishing that

$$\left(1 - \frac{1}{r+1}\right)\gamma_r^* \le c_r < 5\,\gamma_r^*, \qquad \gamma_r^* = \frac{1}{\binom{r}{\lfloor \frac{r}{2} \rfloor}} \asymp \frac{r^{1/2}}{2^r},$$

i.e., that the Stechkin constant c_r, far from increasing with r, does in fact decay exponentially fast. We also show that the same upper bound is valid for the constant $c_{r,p}$ in the Stechkin inequality for L_p-metrics with $p \in [1,\infty)$, and for small r we present upper estimates which are sufficiently close to $1 \cdot \gamma_r^*$.

Lagrange vs. Ideal Interpolation in Several Variables

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In one and two variables every ideal Projector is a limit of Lagrange projectors. In three or more variables it is not so. I will describe the set of those projectors that are the

limits of Lagrange projectors and give a finite steps algorithm for determining whether a given projector is or is not in this set. This provides one possible answer to a question of Carl de Boor. The results are also related to some open problems in Algebraic Geometry.

Interpolation in Special Orthogonal Groups

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Construction of smooth interpolating curves in non-Euclidean spaces is an interesting theoretical problem which finds many applications in engineering and physics. In the present work, we address the question of interpolating points in Lie groups, focusing on a special orthogonal group $SO(n)$ due to its practical importance. Our technique is based on the connection between the group and its Lie algebra of skew-symmetric matrices and the fact that the exponential map is onto. There exists several methods and algorithms which solve the interpolation problem. In the case of the $SO(3)$ group, various re-parametrizations of rotation matrices (e.g. rotation axes and angles, unit quaternions) are adopted and cubic spline interpolation is performed on such representations. Very common are modifications of the De Casteljau algorithm, in which the key idea is to replace linear interpolation by geodesic interpolation. However, many of the existing algorithms are applied only to localized set of data points, which we believe falls short of having an adequate approach, since the interpolation problem becomes particularly interesting and challenging in a global sense. Our goal is to develop a computationally inexpensive multi-purpose algorithm.

q-Bernstein Polynomial Bases of Triangular Domains

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We define a basis of q-Bernstein polynomials and we use it to construct an orthogonal basis for polynomials in q-Bernstein form on triangular domains. We show how the de Casteljau and degree elevation algorithms can be reformulated in terms of these polynomials.

Best interval quadrature formulae for convolution classes

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Let L_1 and C be the spaces of 2π-periodic functions $f : \mathbf{R} \to \mathbf{R}$ endowed with the corresponding norms. For a nonnegative function $f \in L_1$ let us denote by $P(f, t)$

the decreasing rearrangement of the restriction of f to $[0, 2\pi)$. For an arbitrary function $g \in L_1$ set $g_\pm(t) = \max\{\pm g(t); 0\}$ and $\Pi(g, t) = P(g_+, t) - P(g_-, 2\pi - t)$.

Let $F \subset L_1$ be such that $\{f \in F \ : \ f \perp 1\} \neq \emptyset$. The set F is called Π-invariant if conditions $f \in F$ and $\Pi(g) = \Pi(f)$ imply $g \in F$.

We shall denote by $\nu(g)$ the number of sign changes of the functin $g \in C$ on a period. Let $K \in L_1$, and let $\mu = \mu(K) = 1$ if $\int\limits_0^{2\pi} K(t)\,dt = 0$ and $\mu = 0$ in other case. The function K is called *CVD-kernel* if $\nu(a\mu + K * f) \leq \nu(f)$ for every function $f \in C$, $f \perp 1$, and for every $a \in \mathbf{R}$.

Denote by $K * F$ the set of functions of the form $f = a\mu + K * \phi$, where $a \in \mathbf{R}$, $\phi \in F$ and $\phi \perp \mu$.

Let $n \in \mathbf{N}$ and $0 < h < \pi/n$. Denote by Q_n^h the set of interval quadrature formulae of the form

$$\kappa(f) = \sum_{j=1}^{m} a_j/2h \int\limits_{x_j - h}^{x_j + h} f(t)\,dt,$$

where $x_1 < x_2 < \ldots < x_m < x_1 + 2\pi$, $a_j \in \mathbf{R}$, $m \leq n$.

Let $R^\pm(K * F, \kappa) = \sup\{\int\limits_0^{2\pi} (\pm f(t))\,dt - \kappa(\pm f) \ : \ f \in K * F\}$.

Let $Q \subset Q_n^h$. An interval quadrature formula $\bar{\kappa} \in Q$ is called Q-optimal for the class $K * F$ if $|R^\pm(K * F; \bar{\kappa})| \leq |R^\pm(K * F; \kappa)|$ for every formula $\kappa \in Q$.

Let $Q_{n,\sigma}^h$, $\sigma \in \mathbf{R}$, be the set of all interval quadrature formulae for which $\sum\limits_{j=1}^{m} a_j = 2\pi\sigma$. Let $\kappa_{n,\sigma}^h \in Q_{n,\sigma}^h$ be the formula with n equidistant nodes $\{x_j\}$ and equal coefficients.

We have proved the following

Theorem. *Let $n, r \in \mathbf{N}$, $0 < h < \pi/n$; $K \in CVD$ and F be an arbitrary Π-invariant set such that $K * F \subset C$. Let $\sigma \in \mathbf{R}$ be an arbitrary number and $Q = Q_{n,\sigma}^h$ if $\mu = 0$, and $\sigma = 1$, $Q = Q_{n,1}^h$ if $\mu = 1$. Then the formula $\kappa_{n,\sigma}^h$ is the Q-optimal for the class $K * F$.*

Polynomials with Shifts

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Let $v_n \in \Pi_n$ be a polynomial expanded in terms of p_ν,

$$v_n = \sum_{\nu=0}^{n} a_{\nu,n} p_\nu \ ,$$

where the elements of $\{p_\nu \in \Pi_\nu\}_{\nu \geq 0}$ satisfy a three term recurrence relation. By a k-shift of the Fourier- and recurrence coefficients we obtain polynomials

$$v_{n-m}^{[k]} = \sum_{\nu=0}^{n-m} a_{\nu+k,n} p_\nu^{(k)} \in \Pi_{n-m} \ , \quad 0 \leq k \leq m \leq n \ ,$$

which generalize the associated (orthogonal) polynomials $\{p_\nu^{(k)} \in \Pi_\nu\}_{\nu\geq 0}$ of order $k \geq 0$. By suitable combinations of shift- and recurrence strategies it can be seen that they are very useful in many fields of numerical mathematics and in signal processing.

We present some properties and applications of these polynomials. Especially we consider decomposition strategies for large n which can be used for analyzing given frequencies of signals, for evaluation of high degree polynomials, etc.

On the uniqueness and non-uniqueness of minimal projections in L_p

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A projection from X onto V is called minimal if it has smallest possible norm. In L_2 all minimal projections have norm 1 and are unique. The situation is different for $p \neq 2$. Several examples will be given to demonstrate the difference between L_2, L_1, $L_p(1 < p < \infty, p \neq 2)$ and $\ell_p\,(1 < p < \infty, p \neq 2)$. We will present what is known today about uniqueness and non-uniqueness of minimal projections in L_p spaces.

The results presented are joint work with G.Lewicki and B.Shekhtman.

A New Family of Multivariate Macro-Elements: How Geometry Determines the Structure of Spline Spaces

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Macro-elements based on the well-known Clough-Tocher, Powell-Sabin, Worsey-Farin, and Alfeld splits are shown to be a part of a larger multivariate family based on what we call k-plus splits. We prove that certain rather severe geometric constraints (overlooked in Worsey-Farin's construction) need to be imposed to ensure smoothness of the splines.

On the stability of quasi-hierarchical Powell-Sabin B-splines

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QHPS-splines are a hierarchical variant of the classical Powell-Sabin (PS-) splines. They are C^1-continuous quadratic macro elements defined on a (non-conforming) hierarchical triangulation. Such mesh is obtained, starting from a conforming triangulation, by

partitioning recursively a subset of triangles with a triadic split. The construction of the spline is based on a particular refinement of the mesh, analogous to the PS-6-split.

For this spline space a normalized quasi-hierarchical basis can be constructed. The basis retains all the advantages of the classical Powell-Sabin B-splines: the basis functions have a local support, they form a convex partition of unity, and the spline is locally controllable by means of control triangles. Since the mesh is no longer restricted to be conforming, local subdivision can be applied straightforwardly. In general, the basis is weakly L_p-stable, but we will show that the mesh can be locally adapted such that the QHPS-basis on the new mesh is strongly L_p-stable.

Higher Order Derivatives in Image Processing

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We are interested in applications of regularization methods containing second (or higher) order derivatives in image processing. We will show that these techniques can be successfully used in

1. edge-preserving denoising of *scalar-valued* images,
2. simultaneous estimation and decomposition of (*vector-valued*) optical flow,
3. restoration of *tensor-valued* images.

1. Edge-preserving denoising of images. Starting from a drawback of the well-established Rudin-Osher-Fatemi model, namely the so-called staircasing effect,we are interested, *e.g.* in discrete analogons of functionals of the form $\int_\Omega (f-u)^2 + \alpha \|H(u)\|_F \, dx$, where $H(u)$ denotes the Hessian of u. In the one-dimensional setting, the minimizers of the considered functionals are discrete polynomial splines with free knots or inf-convolution splines.

2. Simultaneous estimation and decomposition of optical flow. An important research problem is to develop variational approaches that render flow estimation from image sequences into a well-posed and numerically stable problem, while preserving small-scale flow structures that are important for empirical investigations of turbulent phenomena. We introduce a novel class of variational flow estimation schemes by combining higher-order flow regularization with recent techniques developed for the decomposition of images into structural and textural parts by applying TV and G-norms. As a proper discretization we apply the finite mimetic difference method that preserves the integral identities fulfilled by the continuous differential operators. As a result, we obtain variational approaches that allow not only for estimating fluid flow from image sequences but simultaneously yield a decomposition of the flow into coherent spatio-temporal flow patterns and small-scale structures.

3. Restoration of matrix fields. Recently matrix-valued data sets have gained significant importance in image processing applications, e.g. in medical diffusion tensor magnetic resonance imaging (DT-MRI). We show how successful restoration methods from scalar-valued imaging (see 1.) can be translated to matrix-fields. Here the preservation of the positive definiteness of the matrices and the inclusion of Jordan products of symmetric matrices comes into the play.

This is joint work with S. Setzer, Ch. Schnoerr, J. Yuan (University of Mannheim) and S. Didas (Saarland University).

On the approximation order of the truncated frame operator

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Multiresolution analysis provides the foundation for the construction of wavelets and frames, for both the stationary shift-invariant and the irregular setting. The approximation order of the truncated frame operator, for stationary shift-invariant MRA's, was found to be $m = \min\{2L, m_0\}$ where L is the order of vanishing moments of the frame elements and m_0 is the (maximal) approximation order of the underlying MRA, see [1]. This "doubling" of the number of vanishing moments does not occur, however, for irregular frames and frames on bounded intervals. In this talk, we make use of quasi-projection kernels in order to characterize the approximation order of the truncated frame operator in the univariate case.

[1] I. Daubechies, B. Han, A. Ron, Z. Shen, Framelets: MRA-based constructions of wavelet frames, Appl. Comput. Harm. Anal. 14 (2003), 1-46.

Rate of convergence of iterates of linear operators to semigroups and resolvent operators

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Trotter's theorem (1958) on the approximation of C_0-semigroups has been largely used in the last twenty years in the approximation of the solution of a given evolution problem by means of iterates of suitable approximation processes. An open question has been a quantitative estimate of the convergence which was not directly available for this theorem. Only in the last years some results in this direction has been found by Gonska and Rasa (2006) for Bernstein operators and by Mangino and Rasa (2007) in a general case. The results presented here have been obtained in collaboration with Campiti and hold under the same general assumptions of Trotter's theorem. In particular, we don't make any assumptions on the growth bound of the limit semigroup and compared with the existing literature our estimates are better even in the case of growth bound equal to 0 and behaves correctly on the invariant subspaces for the approximation process.

As a consequence, we are able to apply our results to general approximation processes even in an infinite-dimensional context. As regards to some approximation processes on the simplex we provide quantitative estimates on the space of all functions of class $C^{2,\alpha}$, extending considerably the class C^3 previously considered by Mangino and Rasa.

Finally, we point out the possibility of yielding a quantitative estimate for the resolvent operator, whose description in terms of iterates has been recently obtained by Albanese, Campiti and Mangino (2006).

How Many Random Projections Does One Need to Recover a k-sparse Vector?

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The essential information contained in most large data sets is small when compared to the size of the data set. That is, the data can be well approximated using relatively few terms in a suitable transformation. This paradigm of Compressed Sensing suggests a revolution in how information is collected and processed. In this talk we consider a stronger notion of compressibility, sparsity, which measures the number of non-zero entries. For data sets which are sparse (possibly following a transformation), the data can often be recovered efficiently, with relatively few randomized measurements by utilizing highly non-linear optimization based reconstruction techniques.

Specifically, consider an underdetermined system of linear equations $y = Ax$ with known y and $n \times N$, matrix A with $n < N$. We seek the sparsest solution, i.e., the x with fewest nonzeros satisfying $y = Ax$. In general this problem is NP-hard. However, for many matrices A there is a threshold phenomenon: if the sparsest solution is sufficiently sparse, it can be found by linear programming. Quantitative values for a strong and weak threshold will be presented. The strong threshold guarantees the recovery of the sparsest solution x_o, whereas a weaker sparsity constraint ensures the recovery of the sparsest solution for most x_o. Connections with high-dimensional geometry imply results about the structure of Gaussian point clouds and the neighborliness of polytopes.

On optimal estimators in learning theory

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This talk addresses some problems of supervised learning in the setting formulated by Cucker and Smale. Supervised learning, or learning-from-examples, refers to a process that builds on the base of available data of inputs x_i and outputs y_i, $i = 1, \dots, m$, a function that best represents the relation between the inputs $x \in X$ and the corresponding outputs $y \in Y$. The goal is to find an estimator f_z on the base of given data $z := ((x_1, y_1), \ldots, (x_m, y_m))$ that approximates well the regression function f_ρ (or its projection) of an unknown Borel probability measure ρ defined on $Z = X \times Y$. We assume that (x_i, y_i), $i = 1, \ldots, m$, are indepent and distributed according to ρ.

There are several important ingredients in mathematical formulation of this problem. We follow the way that has become standard in approximation theory and has been used in recent papers. In this approach we first choose a function class W (a hypothesis space H) to work with. After selecting a class W we have the following two ways to go. The first one is based on the idea of studying approximation of the $L_2(\rho_X)$ projection $f_W := (f_\rho)_W$ of f_ρ onto W. Here, ρ_X is the marginal prabability measure. This setting is known as the *improper function learning problem* or the *projection learning problem*. In this case we do not assume that the regression function f_ρ comes from a specific (say, smoothness) class of functions. The second way is based on the assumption $f_\rho \in W$. This setting is known as the *proper function learning problem*. For instance, we may assume that f_ρ has some smoothness. We will give some upper and lower estimates in both settings.

Movement detection using quaternion wavelet phase space decomposition

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We use wavelet analysis to detect movement on a pair of stereographical movies and quaternion wavelet phase space decomposition to match the correspondent images in each picture in order to extract the movement characteristics and approximate the equation of the movement

Sampling on Sparse Grids

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We consider a d-variate periodic, complex valued and continuous function $f : T^d \to C$, where d may be large. Our aim is to approximate f by sampling operators and investigate the error in the L_p-metric. Such a sampling process $\{A_m\}_m$ uses only discrete information about the function f in the following way

$$A_m\, f(x) = \sum_{k=1}^{N_m} f(x_k)\psi_k(x) \quad , \quad x \in T^d \quad ,$$

where the sampling points $x_k \in T^d$ and the functions $\psi_k : T^d \to C$, $k = 1, ..., N_m$, are fixed. We will focus on the so called Smolyak algorithm. Starting with a sequence of sampling operators for the univariate case we obtain, via a special tensor product construction, a sequence $\{A_m\}_m$ of sampling operators for the d-variate case acting on a so called sparse grid. This construction provides useful properties. Having more information about f, for instance, f belonging to some periodic Sobolev space with dominating mixed derivative, we are able to prove $L_p(T^d)$-error estimates depending on the size of the used sampling grid and present new upper bounds for the problem of optimal recovery of functions.

Fixed Point Theorems with Applications to Solutions for Functional Equations Arising in Dynamic Programming

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In this talk, we introduce and study two new functional equations, which contain a lot of known functional equations as special cases, arising in dynamic programming of multistage decision processes. By applying a new fixed point theorem, we obtain the existence, uniqueness, iterative approximation and error estimate of solutions for these functional equations. Under certain conditions, we also study properties of solutions for one of the functional equations. The results presented in this paper extend, improve and unify the results due to Bellman, Bellman and Roosta, Bhakta and Choudhury, Bhakta and Mitra, Liu, Liu and Ume and others. Two examples are given to demonstrate the advantage of our results than existing results in the literature.

Approximation of Intrinsic Mode Functions by Smooth Piece-Wise Constant Amplitude and Phase Functions

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The Intrinsic Mode Functions(IMF) arise as the basic building blocks extracted by applying the Empirical Mode Decomposition(EMD) Method to signals or more general functions. The EMD is adaptive nonlinear method for analyzing nonstationary and nonlinear phases of signals. Recently we characterized IMF's as eigenfunctions of self-adjoint differential operators. In the talk we present a method based on the corresponding differential equations for local approximation of IMF's by elementary functions of the form $e^{at}\cos(bt + c)$, where a is real and $b > 0$. Since for any self-adjoint operator there exists a complete sequence of eigenfunctions the method can be successfully used for decomposing and approximating functions with square integrable second derivatives.

Competitive on-line predicton as approximation problem

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In this talk I will discuss similarities and differences between typical problems considered in approximation theory and learning theory, and then review some results of competitive on-line learning involving Banach benchmark classes.

Subdivision in Lie groups and symmetric spaces

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We show how to define and to analyze curve subdivison schemes operating in Lie groups and symmetric spaces. We especially consider C^2 smoothness.

Wavelet approach to image demosaicing

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Many digital cameras use Bayer pattern to sample red, green, and blue channels of color images. At a location of each pixel, only one color sample is taken and the values of other colors at the location must be recovered. In Bayer pattern the ratio of the samples of R,G, and B is 1/4:1/2:1/4. Recovering color images from their Bayer patters is called demosaic. Based on the fact that the color images have very high inter-chnnel correlations in their high fraquency subbands, we formulate demosaicing as a constrained minimization problem in wavelet domain. The algorithms for solving the problems are presented.

Direct and Inverse Theorems for RBF-type functions: Implications and Applications

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In their book on approximation, DeVore and Lorentz have a chapter on *Central Theorems of Approximation.* In particular they discuss *direct theorems* such as the classical

Jackson theorems for approximation by trigonometric polynomials and the corresponding *inverse theorems* of Bernstein for trigonometric polynomial approximation. In this survey talk, we wish to discuss recent progress concerning both direct and inverse theorems for radial basis functions (RBFs) and spherical basis functions (SBFs). Contributions in this area have been made by several authors. Central to the inverse theorems is an "L_2 Bernstein-type" inequality for certain RBFs and a more general "L_p Bernstein-type" inequality for SBFs.

When an SBF is defined as the restriction to the n-sphere of an associated RBF, direct theorems for the SBF can be derived from the corresponding direct theorems of the RBF. The talk will give reasons for this connection and explain how a similar line of reasoning connects the least squares approximation results of both RBFs and SBFs.

Spherical basis functions were a natural outgrowth of radial basis functions. Similarly, matrix-valued radial basis functions, which allows one to approximate and interpolate vector fields, have also evolved from RBFs. This talk will also include very recent "Sobolev-type" error estimates for these functions as well as an application to modelling certain physical phenomena. Finally, a construction of divergence-free SBFs will be discussed with emphasis on certain applications in geodesy.

Spherical Splines for Hermite Interpolation and Surface Design

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We explain how to use spherical splines over spherical triangulation for surface design. One is to construct spherical splines satisfying Hermite interpolation conditions. Another one is to fill a surface hole. The third one is how to deal with point clouds. Several surfaces are shown to demonstrate our method.

Image reconstruction and orthogonal expansions

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We discuss an algorithm for reconstruction of images from x-ray data. The algorithm is based on orthogonal polynomial expansions on the unit disk. It preserves polynomials of high degree and converges uniformly for functions satisfying mild smoothness condition. We will show by example that the reconstruction is fast and effective.

One-sided L^p Norm and Best Approximation in One-sided L^p Norm

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Let f be a p integrable function on K, a compact subset of R, and μ be a σ-finite positive measure. For $p \geq 1$, the one-sided L^p norm is defined as follows:

$$||f||_p = \max\{(\int_{\{f>0\}} |f|^p d\mu)^{\frac{1}{p}}, \ (\int_{\{f<0\}} |f|^p d\mu)^{\frac{1}{p}}\}$$

We first prove the above definition is indeed a norm. Then the best approximation in the one-sided L^p norms is studied. Among others, characterization and uniqueness of best approximation are discussed.

Hermite Subdivision in Geometric Settings

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We shall discuss how to extend classical Hermite subdivision schemes to the following two geometric settings:

(i) We have a discrete set of initial points and velocity vectors on a Lie group, and we show how to use Hermite subdivision to obtain smooth interpolants of such data.

(ii) We have a 2-simplicial complex with (suitably defined) order r jet data defined at the vertices. And we show how to apply Hermite subdivision to obtain smooth functions defined on the whole simplicial complex, where smoothness is defined w.r.t. to a suitable chosen affine differential structure on the complex. Standard subdivision surfaces used in computer graphics can be viewed as a special case ($r = 0$) of this setting.

Smoothness Equivalence Properties of Subdivision Schemes of Manifold-valued Data

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Interpolation of manifold-valued data is a fundamental problem which has applications in many fields. Linear subdivision method is an efficient and well-studied method for interpolating or approximating real- valued data in a multiresolution fashion. In this talk, we shall recall several natural algorithms for adapting a linear subdivision schemes to manifold-valued data. Afterwards, we present results that show that such a nonlinear subdivision scheme typically yields curves that are as smooth as those generated by the underlying linear subdivision scheme.

Symmetry in basis function methods

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Approximation methods based on positive definite functions are widely used for scattered data problems, for prediction of stationary processes, and in machine learning. In the classical case of the d-dimensional Euclidean space, the basis function is commonly assumed to be radial. This isotropy property can algebraically be described with rotation invariance. Using other groups, leads to further examples of symmetry.

Coming from the Euclidean norm, radiality also has a second generalization. Replacing the Euclidean norm with a p-norm, $1 \leq p \leq \infty$, on the d-dimensional space, leads to another type of symmetry, which only in the case $p = 2$ can be fully described within the algebraic context.

The talk will focus on these different aspects of symmetry, discussing their connection and differences.