Constr. Approx. (1996) 12: 437–438

CONSTRUCTIVE APPROXIMATION

Erratum: Optimal Ray Sequences of Rational Functions Connected with the Zolotarev Problem

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Concerning the numerical methods presented in Section 7 of [1] for generating ray sequences of rational functions r_{mn} that are asymptotically optimal for the Zolotarev problem

$$Z_{mn} = Z_{mn}(E_1, E_2) := \inf_{r \in \mathbf{R}_{mn}} \left(\frac{\sup_{z \in E_1} |r(z)|}{\inf_{z \in E_2} |r(z)|} \right), \qquad \frac{m}{n} \to \lambda,$$

the discussion of Leja–Bagby points in part (b) requires clarification. Indeed, as pointed out by V. Totik, the argument given on pages 259–261 of [1] does not, in general, cover the hypothetical case when the sequence of signed measures $\{\sigma_n\}$ has more than one limit measure. That is, we have actually established in [1] the following result.

Proposition 1. *Suppose the disjoint compacta E_1, E_2 are regular and $\lambda = p/q$ is rational. Set $\tau = \lambda/(\lambda + 1)$ and construct the Leja–Bagby points for the condenser (E_1, E_2) and corresponding rationals $r_n(z) \in \mathbf{R}_{np,nq}$ as in* [1, p. 259]. *If the associated sequence $\{\sigma_n\}_{n=1}^{\infty}$ of signed measures defined in* (7.9) *of* [1] *converges in the weak-star sense to the signed measure μ, then $\mu = \mu^*(\tau)$, where $\mu^*(\tau) = \mu_1^*(\tau) - \mu_2^*(\tau)$ is the extremal* (*equilibrium*) *measure given in* [1, Theorem 3.1]. *Consequently, the ray sequence $r_n(z)$ is asymptotically optimal for the Zolotarev problem* (*i.e., the limit relation following* (7.10) *of* [1] *holds*).

As we now show, the assumption that the full sequence of measures $\{\sigma_n\}_{n=1}^{\infty}$ converges can be removed if we form the Leja–Bagby points for the condenser (E_1^*, E_2^*) instead of (E_1, E_2), where $E_i^* = E_i^*(\tau) := \operatorname{supp}(\mu_i^*(\tau))$, $i = 1, 2$.

Proposition 2. *If the procedure for constructing Leja–Bagby described in Section* 7(b) *of* [1] *is applied on the support sets E_1^* and E_2^* of the extremal measure $\mu^* = \mu^*(\tau)$,*

Date received: May 8, 1996. Date revised: June 13, 1996. Communicated by Doron S. Lubinsky
AMS classification: 41A20, 30C85, 30E10, 31C15.
Key words and phrases: Rational functions, Ray sequences, Zolotarev problem, Logarithmic potential.

then $\sigma_n \to \mu^*$ in the weak-star topology. Consequently, the associated ray sequence of rationals $r_n(z) \in \mathrm{R}_{np,nq}$ is asymptotically optimal for the Zolotarev problem.

Proof. With the notation of [1], we proceed as on page 259 to obtain the estimate

$$(p+q)^{-2}\log\left(\frac{}{A_{n+1}}\right) = \tau \inf_{E_1^*} U^{\sigma_n} - (1-\tau)\sup_{E_2^*} U^{\sigma_n}$$
$$\leq \int U^{\sigma_n}\, d\mu^* = \int U^{\mu^*}\, d\sigma_n = V_\tau,$$

where the last equality follows since $\operatorname{supp}(\sigma_n^{(1)}) \subset E_1^*$ and $\operatorname{supp}(\sigma_n^{(2)}) \subset E_2^*$. The above inequality has the same form as the inequality (7.7) of [1] given for Fekete points, from which the convergence $\sigma_n \to \mu^*$ follows. ■

Remark 1. The above proof shows, more generally, that Leja–Bagby points for the sets

$$K_1(\tau) := \{z \in E_1 : U^{\mu^*}(z) = F_1(\tau)\}, \qquad K_2(\tau) := \{z \in E_2 : U^{\mu^*}(z) = F_2(\tau)\},$$

which contain $E_1^*(\tau)$ and $E_2^*(\tau)$, respectively, have the described convergence properties. Moreover, it can be shown that for τ close to 1/2 (λ close to 1), the Leja–Bagby points for the condenser $(K_1(\tau), K_2(\tau))$ are the same as the Leja–Bagby points for (E_1, E_2) and hence the latter have the desired convergence properties.

Remark 2. For the case of two intervals, we have found that Fejér-Walsh points appear to perform better than the corresponding Leja–Bagby points. However, the procedure for generating a convergent sequence of points can be implemented as follows. From Theorems 8.1 and 8.2, for $E_1 = [a_1, b_1]$ and $E_2 = [a_2, b_2]$, $b_1 < a_2$ and $1 > \tau > 1/2$, we know that $E_1^* = E_1$ and $E_2^* = [a_2, s]$, where $s = s(\tau)$. Since τ can be determined explicitly in terms of s, forming Leja–Bagby points for the pair of intervals $[a_1, b_1]$ and $[a_2, s]$, $a_2 < s < b_2$, yields a sequence that converges (by Proposition 2) to $\mu^* = \mu_\tau^*$.

The general problem of convergence of Leja–Bagby points for ray sequences on the condenser (E_1, E_2) remains an open problem.

Reference

A. L. Levin, E. B. Saff (1994): *Optimal ray sequences of rational functions connected with the Zolotarev problem*. Constr. Approx., **10**:235–273.

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